Uncovered Interest Parity (UIP)
The UIP puzzle redux

Giancarlo Corsetti
University of Cambridge
F510 International Finance, Lent 2020
1. Derivation of the (textbook) UIP
   - Asset pricing and the UIP condition
   - Inflation variability and the ‘Siegel paradox’

2. Empirical Test: the Fama regression
   - The joint hypotheses underlying the Fama regression
   - Regression model

3. Understanding the UIP puzzle: textbook vs. new views
   - The UIP puzzle: Forecast horizon
   - The UIP failure switches sign in crises: A ‘new Fama puzzle’?
   - The ‘new puzzle’ is at least one-hundred year old

4. The UIP puzzle redux: it combines at least three pieces of evidence!
1. Derivation of the Uncovered Interest Parity

Consider an uncovered position in foreign currency. Without loss of generality, think of an investor who borrows one unit of foreign currency (say yen) at the rate \((1 + i^*)\), and invest the domestic currency equivalent, \(E_t\), in domestic currency at the rate \((1 + i)\).

Since the position is unhedged, at maturity, the investors will have to acquire foreign currency at the spot rate to repay its debt, i.e. its liability will be \((1 + i^*)E_{t+1}\).

The cash flow in domestic currency from the uncovered position will be \(X_{domestic\,-\,currency} = (1 + i)E_t - E_{t+1}(1 + i^*)\). It is convenient to normalize this cash flow dividing through by the gross foreign interest rate:

\[
X = \frac{1+i}{1+i^*}E_t - E_{t+1}.
\]

By this normalization, if the CIP condition holds, the uncovered position above is equivalent to sell foreign currency forward, generating the cash flow

\[
X = F_t - E_{t+1}
\]

The “Uncovered interest parity hypothesis” boils down to the following condition:

\[
E_t X = 0 \Rightarrow F_t = E_t E_{t+1}
\]
The UIP hypothesis

In its simplest formulation, the UIP hypothesis states that nominal interest rate differentials across currencies should forecast expected exchange rate movements: investing in a currency that is expected to lose value over, say, the next three months should require higher three month nominal interest rates:

\[
\frac{1 + i}{1 + i^*} = \frac{E_t(E_{t+1})}{E_t}. \tag{1}
\]

If the CIP condition holds, the above is equivalent to say that the Forward rate should equal to the expected future spot rate

\[
F_t = E_t (E_{t+1}) \tag{2}
\]

where the forward contract is for delivery at \( t+1 \).

We will now show that the above is an exact equilibrium condition in the assets market only under these assumptions:

(a) risk neutrality and
(b) \( \text{Cov}_t \left( \frac{P_t}{P_{t+1}}, E_{t+1} \right) \approx 0 \)
We have seen already that the cash flow from selling one unit of foreign currency (dollar) forward, is $X = F_t - E_{t+1}$

- $X > 0$ when the dollar spot is weaker than the forward rate ($E_{t+1} < F_t$): the party who sold dollars forward can buy them cheaper than $F_t$ in the spot market for delivery.

Since $X$ can be negative and losses can in principle be quite large, commercial banks deal with the risk of insolvency by screening costumers, and sometimes by requiring a margin. Besides these margins, there is no cash exchange until the maturity of the contract.

Ignoring margins, applying our pricing equation $Q = E_t \left[ \mathcal{D} \frac{P_t}{P_{t+1}} X \right]$ yields:

$$0 = E_t \left[ \mathcal{D} \frac{P_t}{P_{t+1}} (F_t - E_{t+1}) \right]$$
In equilibrium, the forward rate is equal to the expected future spot rate, plus a term capturing more than the risk premium:

\[ \mathcal{F}_t = \frac{E_t \left[ \mathcal{D} \frac{P_t}{P_{t+1}} \mathcal{E}_{t+1} \right]}{E_t \left[ \mathcal{D} \frac{P_t}{P_{t+1}} \right]} = E_t(\mathcal{E}_{t+1}) + \frac{\text{Cov}_t \left[ \mathcal{D} \frac{P_t}{P_{t+1}}, \mathcal{E}_{t+1} \right]}{E_t \left[ \mathcal{D} \frac{P_t}{P_{t+1}} \right]} \]  

Indeed, note that under risk neutrality, that is, \( \mathcal{D} \) is non-stochastic (for simplicity set it equal to 1), the covariance term does not vanish:

\[ \mathcal{F}_t = \frac{E_t \left[ \frac{P_t}{P_{t+1}} \mathcal{E}_{t+1} \right]}{E_t \left[ \frac{P_t}{P_{t+1}} \right]} = E_t \mathcal{E}_{t+1} + \frac{\text{Cov}_t \left[ \frac{P_t}{P_{t+1}}, \mathcal{E}_{t+1} \right]}{E_t \left[ \frac{P_t}{P_{t+1}} \right]} \]

The forward rate is equal to the expected future spot rate \textit{plus} a term in the covariance between the future exchange rate and the inverse of the inflation rate.
Deriving the UIP from equilibrium pricing

The compensation for inflation variability: Jensen’s inequality

- Under risk neutrality **covariance term is the analog of a risk premium but has nothing to do with risk.** Think of it as a ‘compensation for inflation variability.’
  - Suppose that $\mathcal{E}_{t+1}$ can take two values, 1 or 3, with prob. 5. Also, suppose that the price $P_{t+1}$ level moves in tandem with $\mathcal{E}_{t+1}$, that is, $P_{t+1} = \mathcal{E}_{t+1}$.
  - We know that, under risk neutrality, $\mathcal{F}_t = 2$ should lead to $E(x) = 0$, where the cash flow is in real terms. Would this condition be satisfied if risk neutral agents priced $\mathcal{F}_t = E_t \mathcal{E}_{t+1} = 2$?

Let’s calculate the expected return from the contract **in real terms.** Setting $\mathcal{F}_t = 2$ and dividing by the price level:

$$E \left( \frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right) = .5 \left( \frac{2 - 1}{1} \right) + .5 \left( \frac{2 - 3}{3} \right) = .333 > 0!$$

This shows that, if $\mathcal{F}_t = 2$, $E(x) > 0!$

- To ensure that $E(x) = 0$, $\mathcal{F}_t$ must satisfy

$$ .5 \left( \frac{\mathcal{F}_t - 1}{1} \right) + .5 \left( \frac{\mathcal{F}_t - 3}{3} \right) = 0 $$

The solution is $\mathcal{F}_t = 1.5$. Note: $\mathcal{F}_t < E_t \mathcal{E}_{t+1}$ is exactly what you expect when $\text{Cov}_t \left[ \frac{P_t}{P_{t+1}}, \mathcal{E}_{t+1} \right] < 0$, as in our example.
In practice, unless inflation variability is very high (as is in the previous example), there is no much loss in setting $\text{Cov}_t (P_t/P_{t+1}, \mathcal{E}_{t+1}) \approx 0$.

Suppose $P_{t+1}$ can be either 1.45 or 1.55—a range of variation that is still quite high for prices. It is easy to see that $F_t$ becomes arbitrarily close to 2, namely, the following:

\[
.5 \left( \frac{F_t - 1}{1.45} \right) + .5 \left( \frac{F_t - 3}{1.55} \right) = 0
\]

is solved for $F_t = 1.97$.

It is nonetheless important to remember that, whether or not they are risk neutral, agents care about expected cash flows in real terms, not in nominal terms.

Ignoring this point raises all kinds of odd issues, such as the Siegel's paradox.
Ignoring the correction for inflation creates confusion

The so-called Siegel Paradox

If one erroneously believes that $F_t = E_t (E_{t+1})$, then trivially:

$$\frac{1}{F_t} = \frac{1}{E_t (E_{t+1})}$$  \hfill (4)

Now, instead of considering pounds per dollar, consider the reciprocal, dollar per pounds. Then, by changing the definition of exchange rate, it should also be true that:

$$\frac{1}{F_t} = E_t \left( \frac{1}{E_{t+1}} \right)$$  \hfill (5)

The so-called Siegel’s paradox: by Jensen’s inequality, (4) and (5) above cannot hold simultaneously, since

$$E_t \left( \frac{1}{E_{t+1}} \right) \neq \frac{1}{E_t (E_{t+1})}.$$

If the Forward is an unbiased forecast of the future exchange rate when we look at dollars per pounds, the same cannot be true if we consider pounds per dollar!

What is going on here?
An odd implication of ignoring the correction for inflation
The so-called Siegel Paradox

- There is no ‘paradox’ if we start from the correct zero profit condition, expressing the cash flow in real terms. To see this, note that the price level expressed in UK pounds can be translated into dollars by dividing $P$ by $\mathcal{E}$. To express the domestic price level in units of foreign currency, hence, just write: $P^*_{t+1} = P_{t+1}/\mathcal{E}_{t+1}$. Then:

$$0 = E_t \left( \frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{P_{t+1}} \right) =$$

$$= E_t \left( \frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{\mathcal{E}_{t+1}P^*_{t+1}} \right) = E_t \left( \frac{1}{\mathcal{F}_t} - \frac{1}{\mathcal{E}_{t+1}} \cdot \frac{\mathcal{F}_t - \mathcal{E}_{t+1}}{\mathcal{E}_{t+1}P^*_{t+1}} \right)$$

$$= E_t \left( \frac{1}{\mathcal{F}_t} - \frac{1}{\mathcal{E}_{t+1}} \cdot \frac{1}{P^*_{t+1}/\mathcal{F}_t} \right) = \mathcal{F}_t \cdot E_t \left( \frac{1}{\mathcal{F}_t} - \frac{1}{\mathcal{E}_{t+1}} \right) \Rightarrow$$

$$0 = E_t \left( \frac{1}{\mathcal{F}_t} - \frac{1}{\mathcal{E}_{t+1}} \right)$$
2. Empirical Test of the UIP: the Fama regression

In the empirical literature, most tests are derived under the joint null hypothesis

1. UIP: \[ \frac{1+i}{1+i^*} = \frac{F_t}{\hat{E}_t} = \frac{1}{\hat{E}_t} \hat{E}_t \hat{E}_{t+1} \]

2. Rational Expectations (RE)

To derive a regression model under this joint null (UIP+RE), rewrite the exchange rate at \( t+1 \) as the product of its expected value and a forecast error term (exponential):

\[ \hat{E}_{t+1} = \hat{E}_t \hat{E}_{t+1} \cdot \exp(-\nu_t) \]

Note that the error term \( \exp(-\nu_t) \) takes values above or below 1 depending on whether \(-\nu_t\) is positive or negative.

Denoting logarithms by small-case letters, i.e. \( \ln F_t = f_t \), take logs of the above and rearrange:

\[ \ln \hat{E}_t \hat{E}_{t+1} = e_{t+1} + \nu_t \]
The Fama regression

Rewrite the UIP in log form

\[ \ln E_t E_{t+1} - e_t = (f_t - e_t). \]

and substitute the expression for \( \ln E_t E_{t+1} \) above. We obtain:

\[ e_{t+1} - e_t = (f_t - e_t) - \nu_t \]

\( \nu_t \) pure error term (i.i.d.)

Under rational expectations RE, agents

- do not make systematic mistake — hence on average the error must be zero: \( E_t \nu_t = 0 \)
- use all available information to form their forecast — \( \nu_t \) cannot be correlated to any variable known by investors at time \( t \), and must be serially uncorrelated.

In words: under joint null hypothesis of UIP (in turn combining (1) risk neutrality, (2) \( \text{Cov}_t (P_t/P_{t+1}, E_{t+1}) \approx 0 \), and (3) CIP), and RE, the forward rate should embody all the relevant information for forecasting the exchange rate.
The Fama regression

The regression model takes either one of the following forms:

\[ e_{t+h} - e_t = a_0 + a_1 (f_{t,t+h} - e_t) + \nu_t \] (6)

\[ e_{t+h} - e_t = a_0 + a_1 (i_{t,h} - i^*_{t,h}) + \nu_t \] (7)

where \( h \) is the forecast horizon (=maturity of forward contracts and assets). The hypothesis to be tested is \( a_0 = 0, \ a_1 = 1 \) and \( \nu_t \) zero mean and serially uncorrelated.

- In assessing the UIP puzzle, be aware that the R-squared of the regressions is minuscule — the adjusted R-squared is often negative. In other words, the proportion of total variation explained by the regression is very small. The variance of exchange rate changes is many times larger than the variance of interest rate differentials.

The test does not (necessarily) reject UIP either (a) at long investment horizons or (b) at very high frequencies (e.g. 5 minutes), see e.g. Chaboud and Wright JIE 2005. In general, it is rejected—hence the ‘puzzle’.
Given the strong assumptions employed to derive the test, it may not be totally surprising that results do not support the null hypothesis exactly. What is surprising is the type and extent of the empirical failure(s). In this lecture, we focus on two important empirical dimensions of the puzzle:

1. Traditionally, the FAMA puzzle is referred to the fact that the OLS estimates of the coefficient $a_1$ tend to be significantly negative

$$\frac{\text{Cov}(f_{t,t+h} - e_t, e_{t+h} - e_t)}{\text{Var}(f_{t,t+h} - e_t)} < 0$$

and close to -1 for international currencies such as the U.S. dollar, the yen, the Swiss franc, the euro (and before European monetary unification, the D-mark and the franc), at horizons up 4-5 year.

2. In the years after the global crisis or, more in general, after crisis periods, new strong evidence evidence suggests that $a_1$ tends to be significantly larger than one, values of 10 or 20 are not uncommon.
Let's focus first on the view of the puzzle shared before the global financial crisis.

- A negative $a_1$ means that, **an increase in the Home nominal interest rate relative to the Foreign one at $t$** tends to be systematically associated with (forecasts) **appreciation of the Home currency between $t$ and $t+h$**.

- The first graph to follow shows averages of the estimates of $a_1$ from a large set of regressions for many currency pairs, using interest rates and exchange rates at different horizons: $h=3,6,12$ months, 5 and 10 years (see Chinn and Meredith 2005, Chinn and Quayyam 2012 and Valchev 2015).

The UIP puzzle:
Forecast horizons

Average coefficient, different time horizons

![Bar chart illustrating the average coefficient for different time horizons: negative for short horizons, positive for long horizons.](chart.png)

From the recent literature (Chinn and Meredith, 2005, Chinn and Quayyam, 2012, Valchev, 2015)

(i) Negative estimated $\beta$ for short horizons
(ii) Interest differential point to the right at longer horizons

See Chinn and Zhang (2015) for an explanation using a NK DSGE model.
The UIP puzzle: Forecast horizons


The figure report estimates and confidence intervals.
The UIP failure switches sign in crises: A ‘new Fama puzzle’?

- Recently, we have become aware that UIP failure can take a different sign. After the global crisis, the estimated coefficient $a_1$ becomes positive and exceed 1.

- Interest differentials forecast “excessive,” i.e., “more than proportional” expected depreciation.

- The following graphs document this shift. They are from Brussiere Chinn Ferrata and Heipertz 2015 and Ca’ Zorzi and Marin 2017.
A ‘new Fama puzzle’?
Positive and large coefficients in the aftermath of crises

Estimated coefficient $a_1$ from Fama regression run over 3 year rolling windows, 12 month horizon.
The coefficient is clearly positive and very high after 2007, but also in 1997-98 (the crisis in South-East Asia).
The Fama coefficient before the global financial crisis

The coefficient before 2007

Estimated coefficient $a_1$ from Fama regression run over 3 year rolling windows, 3 and 12 month horizon, sample 1986-2006 (Dec).

![Graph showing estimated coefficients for different countries and horizons before and after the GFC](image-url)
Estimated coefficient $a_1$ from Fama regression run over 3 year rolling windows, 3 and 12 month horizon, since 2007-2011.
The new puzzle is one-hundred year old
1930s’, mid 1990s and the Great Recession compared

GBP-US historical data (Emile Marin): compare the Great Depression with Great Recession.

![Rolling windows regression - 3M maturity (window = 60 months; step = 1 month)](image_url)

Giancarlo Corsetti (Uni of Cam)
The ‘UIP puzzle’ redux

We conclude this notes by reformulating what the puzzle is, combining the evidence on deviations from the null hypothesis of UIP. The coefficient on the interest differentials in the Fama regressions is not equal to one, but:

1. significantly below unity and typically non-positive for investment horizons up to 5 years, as stressed by the textbook treatment of the UIP puzzle in normal times;
2. positive, and above unity during crises;
3. at any point in time, closer to unity for long (as opposed to short and medium-term) investment horizons during both normal and crisis times. (Fama coefficients are much less variable, but not necessarily equal to one, at longer investment horizons.)

whereas normal times are defined as periods in which business cycle movements are neither associated with financial crises, nor give rise to abnormally large and persistent downturns.
References

- Chaboud A.P. and J. Wright “Uncovered interest parity: it works, but not for long”, Journal of International Economics, 2005, 66(2) 349-362