Exchange Rate Misalignment, Capital Flows and Optimal Monetary Policy Trade-offs

Giancarlo Corsetti  
*Cambridge University and CEPR*  
Luca Dedola  
European Central Bank and CEPR  
Sylvain Leduc  
*Bank of Canada*

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Abstract

How should monetary policy respond to inefficient capital flows that cause exchange-rate misalignments and distort current account balances? Using the workhorse open-macro monetary model, we derive a quadratic approximation of the utility-based global loss function, and solve for the optimal targeting rules under cooperation. The optimal response to inefficient inflows exacerbates misalignments when exchange-rate pass-through (ERPT) is incomplete—leans against misalignments otherwise. We show analytically that, when capital inflows lead to over-appreciation, the optimal stance is contractionary and lowers inflation, under incomplete ERPT. It is expansionary if ERPT is complete, reducing competitiveness losses at the cost of inefficiently higher inflation.

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1 Introduction

Inefficient capital flows that distort current account positions and raise concerns about exchange rate misalignment confront monetary policymakers with challenging trade-offs between internal objectives (inflation, and output gap) and external objectives (competitiveness and trade imbalances). A leading thesis holds that "an external deficit raises the natural rate of interest," hence should be matched by a tighter monetary stance (see, e.g., Obstfeld and Rogo[2010]). In actual experience of policymaking, however, external deficits appear to have been factored into monetary policy decisions in different ways. By way of example, the persistent current account deficits that Germany run in the 1990s, were among the reasons why the Bundesbank adopted a contractionary monetary stance in the aftermath of German unification, during which capital inflows into the country translated into a steep appreciation of the D-mark. Conversely, when the global “Saving Glut” (Bernanke [2005]) at the end of the same decade started to cause the dollar exchange rate to appreciate and the US current account to deteriorate, the US monetary authorities maintained an accommodative monetary stance.2

In this paper we reconsider the monetary trade-off between internal and external objectives by studying inefficient capital flows in the workhorse open economy monetary model with incomplete markets—the two-country New Keynesian framework in which the only internationally traded asset is a non-contingent bond (as in the seminal contribution by Obstfeld and Rogo[1995]; see also Costinot et al. [2015] and Dávila and Korinek [2018]).3 In the presence of financial market imperfections, however, the natural rate allocation is not necessarily a desirable compass for stabilization policy—a point recently stressed also by Farhi and Werning [2016]. Because shocks are not fully insurable in a bond economy (the same core financial market distortion analyzed in Costinot et al. [2015]), pecuniary externalities imply that the real exchange rate is misaligned independently of nominal rigidities. As a result, the valuation of current and future national outputs is distorted, and so are the incentives to borrow and lend across borders. Exchange rate movements drive differences in national wealth by affecting the present discounted value of a country’s output (namely, the natural borrowing constraint in a bond economy), similarly to their valuation effects on outstanding foreign assets and liabilities already stressed by the literature (see, e.g., Gourinchas and Rey 2014). Because it affects the relative value of output when financial markets are incomplete, real exchange rate misalignment induces an inefficient wealth wedge across countries. This is where our paper complements the literature focusing on misalignment due to nominal rigidities, that distort demand allocation across countries without impinging on relative wealth (see Engel [2011]).4

Our main contribution consists of providing an analytically transparent characterization of

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1 “Better macro performance comes from a monetary rule that recognizes how an external deficit raises the natural real rate of interest,” Obstfeld and Rogo [2010] p. 34.
2 Systematic evidence on the monetary response to capital inflows is scarce. Kaminsky, Reinhart and Vegh [2004] document expansionary macroeconomic policies during episodes of capital inflows. For the monetary policy implications of the US current account, see Ferrero, Gertler, and Svensson [2009].
3 In the tradition of Obstfeld and Rogo [1995], we capture the lack of efficient diversification in the data despite the number of seemingly available cross-border assets, by focusing on bond economies. However, we do not restrict preferences to have a unit intratemporal elasticity, unlike Clarida, Galí and Gertler [2002] and Engel [2011].
4Cross-country consumption misallocation is proportional to real exchange rate misalignment under complete asset markets. In the terminology of Farhi and Werning [2016], we thus study economies with both aggregate demand externalities, due to nominal rigidities, and pecuniary externalities, due to incomplete asset markets.
the monetary trade-offs raised by inefficient capital flows, showing that the sign and strength of the optimal monetary stance depend on structural features that are inherent to open economies: the degree of exchange rate pass-through (henceforth ERPT) and the sensitivity of output and aggregate demand to the real exchange rate. The role played by these features in shaping the optimal monetary stance squares with basic economic intuition. In general, across all the economies we consider, the optimal response to an inefficient current account deficit is not necessarily contractionary, nor does it follow the natural rate compass. Rather, we show that—provided that the price elasticity of exports exceeds a threshold below unity—the optimal monetary response to a capital inflow depends on whether exchange rate movements have little impact on competitiveness and the output gap (due to low ERPT) or, instead, ERPT is complete. To wit: an inefficient capital inflow that over-appreciates the Home currency calls for a tightening of Home monetary policy under low ERPT, above and beyond the stance dictated by the natural rate allocation, and thus resulting in a fall in Home consumer price CPI inflation. Conversely, it is optimal to pursue a monetary expansion that mitigates the real exchange rate overvaluation, despite its inflationary effects, when competitiveness and output are sensitive to misalignment, due to complete ERPT. These results have significant implications for exchange rate volatility and external imbalances. With incomplete markets, the optimal monetary stance will tend to raise exchange rate volatility in economies where import and export prices do not fully adjust to exchange rate movements, relative to economies characterized by a high degree of ERPT.

Two results are worth stressing. First, the paper provides a second-order accurate approximation of the global welfare function for the standard New Keynesian two-country model with incomplete markets. The welfare function encompasses different models of ERPT—with export prices being sticky either in the currency of the producers (producer currency pricing or PCP, whereas ERPT is complete) or in the currency of the destination market (local currency pricing or LCP, whereas ERPT is incomplete). The derivation of this function does not rely on specific forms of market incompleteness (e.g., bond economies and financial autarky obtain as special cases), nor on restrictive assumptions about preferences (e.g., it is not restricted to the case of unitary trade elasticity). We show that in addition to output gaps and inflation rates, the arguments of the welfare function include real exchange rate misalignment and relative demand misallocation, themselves a function of inefficient capital flows. Different from the case of complete markets, where misalignment and demand misallocation are proportional to each other, these distortions, combined, define a new gap, which we dub “wealth gap” that turns out to play a key role in optimal policy design.

Second, the paper derives optimal targeting rules under cooperation and commitment for PCP and LCP economies. These rules hold for a wide range of shocks (including anticipated or unanticipated shocks to preferences, productivity, markups, etc.), but, unlike the global welfare function, are specific to bond economies. Based on these rules, the paper provides a transparent analytical characterization of macroeconomic dynamics under the optimal monetary policy in response to inefficient capital flows. From the vantage point of monetary policy-making, inefficient capital flows open a wealth gap that acts much like an endogenous “markup” shock—

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5We focus here on the two symmetric cases of ERPT, which have been so far center stage in the literature on the optimal design of monetary policy in open economies, see Engel [2011]. In ongoing work we analyze the key asymmetric case of dominant currency pricing (DCP) recently emphasized by Gopinath [2016], which require a separate, systematic analysis also under complete markets.
raises trade-offs between inflation, output gaps, demand misallocation and misalignment. Most crucially, the wealth gap is a direct synthetic measure of the distortions associated with inefficient flows. The theoretical underpinning of this result resonates with the classical controversy on the transfer problem—originally debated by Keynes and Ohlin concerning the war reparation imposed on Germany after World War I, and more recently reconsidered in the debate on current account rebalancing (see, e.g., Obstfeld and Rogoff [2005]). We show that the wealth gap measures the effect on relative wealth of the transfer implicit in capital flows. Depending on the type of shocks and structural features of the economy—exports price (trade) elasticities and openness—, excessive inflows of capital may actually depreciate (rather than appreciate) the domestic currency and depress aggregate demand, turning the wealth gap negative. We show that when trade elasticities fall short of given thresholds (all well below unity), the optimal monetary response is unambiguously expansionary, irrespective of the degree of ERPT, in order to sustain domestic activity associated with a negative wealth gap. In these economies, even though depreciation discourages capital inflows, the boost to domestic demand from the optimal monetary expansion ends up raising borrowing from abroad, up to the point of increasing the equilibrium external imbalances.

In our analysis of macroeconomic dynamics under the optimal monetary policy, we find it analytically convenient to focus on “news shocks” (anticipation of future changes in fundamentals) as these typically generate capital flows that are excessive relative to the first best. The news shocks may stem from political risk (i.e., capital controls; see, e.g., Acharya and Bengui [2016]), changes in the efficiency of financial intermediaries (see, e.g., Gabaix and Maggiori [2015]), changes in technology or preferences impinging on savings—without loss of generality, we focus on the latter. In the first-best (complete market and flex-price) allocation, even though households are forward looking, relative prices and quantities depend only on the current-period (exogenous) fundamentals, not on their expected realizations in the future—in line with the well-known results in Barro and King [1984]. Relative to this benchmark, under incomplete markets, the entire cross-border flow of capital that responds to news shocks is inefficient. Remarkably, we show that in model specifications often adopted by the literature (see, e.g., Clarida et al. 2002), capital flows in response to news shocks are exogenous to monetary policy and macroeconomic adjustment. We can thus bring our analysis to bear directly on a case often debated in policy circles, where monetary policy can only mitigate the effects of inefficient capital flows on domestic macroeconomic dynamics, but cannot curb their size.

**Literature**  
Our analysis builds on a vast body of work that, over the last two decades, has re-examined a classic question in open economy macroeconomics, concerning the trade-offs between external and internal objective (see Benigno and Benigno [2003]; Clarida, Galí and...
Gertler [2002]; Corsetti and Pesenti [2005]; Devereux and Engel [2003]; Engel [2011]; and Galí and Monacelli [2005], among others.\(^9\) It is nonetheless useful to emphasize two strands of this literature that help highlight our contribution.

The first is the literature epitomized by Engel [2011], who studies optimal policy under both LCP and PCP in the otherwise canonical open economy New Keynesian model developed by Clarida, Galí and Gertler [2002]. A key result stressed by Engel [2011] for LCP economies is that monetary policy can support a constrained-optimal allocation with CPI-price stability and no exchange rate misalignment—which also implies no cross-country demand gaps (as defined in Section 3.1 below). Indeed, under the maintained assumption of complete markets, real exchange rate misalignment and the cross-country demand gap are always proportional to each other—indeed, whether ERPT is complete (PCP) or incomplete (LCP).\(^10\) This proportionality fails under imperfect risk sharing; as shown in our analysis, monetary policy will not be able to close misalignment and demand gaps simultaneously. Still, ERPT will be crucial in determining the extent to which the exchange rate misalignment is stabilized. In particular, under LCP, optimal monetary policy exacerbates misalignment.

The second strand of the literature includes a small number of contributions that, like ours, provide analytical characterizations of the optimal monetary policy in two-country models with incomplete financial markets.\(^11\) Obstfeld and Rogoff [2003] and Devereux [2004] examine static frameworks without capital flows, and in which prices are set one period in advance—therefore, necessarily abstracting from the welfare implications of current account dynamics and inflation. Devereux and Sutherland [2008] study a dynamic setting similar to ours, but in which markets are effectively complete under flexible prices so that price stability also attains the first-best natural rate allocation.\(^12\) Under PCP, Benigno [2009] emphasizes deviations from price stability, in economies in which net foreign asset holdings are asymmetrical in the nonstochastic steady state. However, the focus is on economies in which deviations from both purchasing power parity (PPP) and the law of one price are assumed away, in contrast with the analysis of real exchange rate misalignment at the core of optimal policy design analyzed in our paper. Our paper is also closely related to Farhi and Werning [2016], which provides a general characterization of optimal targeting rules in economies with nominal rigidities and financial market frictions. While in their contribution these authors stress the role of macroprudential policies when monetary policy is constrained, we focus on optimal monetary policy when macroprudential policies are not available. An integral part of our analysis (and a complement to theirs) consists of taking into account the standard welfare costs of inflation studied in the optimal monetary policy literature, that stem from staggered price setting.

Monetary policy with incomplete financial markets is the focus of recent numerical analyses by Rabitsch [2012], who revisits the benefits from international cooperation, and Senay and

\(^9\)As discussed in Corsetti, Dedola, and Leduc [2010], most of the papers in the literature either assume complete markets or close to efficient capital flows because of particular restrictions on preference and technology parameters.

\(^10\)The result also holds when ERPT is asymmetric across borders—the case of DCP recently emphasized by Gopinath [2016]. Casas et al. [2016] study optimal monetary policy for this case, focusing on a small open economy.

\(^11\)Other contributions have looked at similar issues in a small open economy framework—see e.g. De Paoli [2009].

\(^12\)Tille [2005] assesses the welfare impact of integrating international asset markets with nominal rigidities and a stochastic component in monetary policy.
Sutherland [2016], who study the properties of optimal rules in a incomplete markets model with a portfolio of assets including bonds and equities.\textsuperscript{13}

Our study is naturally related to recent literature that emphasizes the role of pecuniary externalities under collateral constraints, financial accelerator (balance-sheet) effects and over- and underborrowing relative to the (constrained-) efficient allocation (see Benigno et al. [2010]; Bianchi [2011]; Bianchi and Mendoza [2010]; Brunnermeier and Sannikov [2015]; Costinot et al. [2015]; Dávila and Korinek [2018]; Jeanne and Korinek [2010]; and Lorenzoni [2008], among others).\textsuperscript{14} Devereux and Yu [2016] characterize optimal monetary policy under discretion in a small open economy with occasionally binding borrowing constraints. Relative to these papers, our contribution considers a standard framework with natural borrowing constraints on short-term debt, which, in equilibrium, respond to both exogenous shocks and the endogenous real appreciation created by inefficient capital inflows—indeed an appreciation relaxes the natural borrowing constraint, by raising the international value of (present discounted) domestic output. A distinct feature is our specific focus on monetary policy in a global equilibrium characterized by overborrowing (and obviously underborrowing in the other country) with respect to both the first-best and the constrained-efficient allocation.

Last, but not least, our results are in line with Woodford [2009], showing that openness to foreign capital does not compromise monetary control, i.e., the ability of the central bank to pursue a desired monetary stance. Yet, as stressed by Rey [2013] and Farhi and Werning [2016], inefficient capital flows may create adverse trade-offs across policy goals, hampering a central bank’s ability to maintain the economy on an efficient path. We complement these papers in that we inspect the monetary policy trade-offs created by capital flows, and characterize the optimal monetary response that can provide a fungible first-line defence in the absence of other readily implementable measures, or complement other policy instruments—ranging from macroprudential policy to capital controls—when these are in place.

The rest of the paper is organized as follows. The next section synthetically goes over the standard two-good, two-country, New Keynesian model that we take as the framework for our analysis. Section 3 derives the global loss function, discussing each of its arguments in some detail, and characterizes the cooperative optimal targeting rules under PCP and LCP. To characterize the optimal policy as transparently as possible, Section 4 focuses on a baseline specification that we dub the Cole and Obstfeld (CO) economy, following Cole and Obstfeld [1991]. This baseline specification sets a unitary trade elasticity, complemented by the assumption of log consumption utility and a linear disutility of labor (the latter is relevant for tractability in the LCP case, as importantly shown by Engel [2011]). In Section 5, we generalize our results to the case of non-unitary trade elasticity, showing that the analytical characterization of the optimal monetary stance in the CO economies still provides tight guidance for policy analysis for sufficiently large trade elasticities.

\textsuperscript{13}A number of other papers numerically solve open economy models under incomplete markets, and examine optimal policy often using ad hoc loss functions. See, for example, Kollmann [2002].

\textsuperscript{14}Cavallino [2016] examines foreign exchange interventions as a second instrument (in addition to conventional interest rate policy) available to the central bank to redress inefficient capital flows in an economy with borrowing constraints similar to those of Gabaix and Maggiori [2015].
2 The model economy

The analysis builds on the standard open economy version of the workhorse model in monetary economics (see, e.g., Clarida, Galí and Gertler [2002] and Engel [2011]), with well-known characteristics. The world economy consists of two countries of equal size, \( H \) and \( F \). Each country specializes in one type of tradable good, produced in a number of varieties or brands defined over a continuum of unit mass. Brands of tradable goods are indexed by \( h \in [0, 1] \) in the Home country and \( f \in [0, 1] \) in the Foreign country. Firms producing the goods are monopolistic supplier of one brand only and use labor as the only input to production. These firms set prices either in local or producer currency units and in a staggered fashion as in Calvo [1983]. Asset markets are complete at the national level, but incomplete internationally.

In what follows, we describe our set-up focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy—variables referring to Foreign firms and households are marked with an asterisk.

2.1 The household’s problem

2.1.1 Preferences

We consider a cashless economy in which the representative Home agent maximizes the expected value of her lifetime utility, where instantaneous utility \( U \) is a function of a consumption index, \( C \), and (negatively) of labor effort \( L \), specialized as follows:

\[
U [C_t, L_t] = \zeta_{C,t} C_t^{1-\sigma} L_t^{1+\eta} \quad \text{where} \quad \sigma, \eta > 0
\]

whereas the model also allows for shocks to marginal utilities of consumption \( \zeta_{C,t} \). Foreign agents’ preferences are symmetrically defined. Households consume both domestically produced and imported goods. We define \( C_t(h) \) as the Home agent’s consumption as of time \( t \) of the Home good \( h \); similarly, \( C_t(f) \) is the Home agent’s consumption of the imported good \( f \). We assume that each good \( h \) (or \( f \)) is an imperfect substitute for all other goods’ varieties, with constant elasticity of substitution \( \theta > 1 \):

\[
C_{H,t} \equiv \int_0^1 C_t(h) \frac{\theta^{-1}}{\phi} dh \quad \text{and} \quad C_{F,t} \equiv \int_0^1 C_t(f) \frac{\theta^{-1}}{\phi} df
\]

The full consumption basket, \( C_t \), in each country, aggregates Home and Foreign goods according to the following standard CES function:

\[
C_t \equiv \left[ a_H^{1/\phi} C_{H,t}^{\phi-1} + a_F^{1/\phi} C_{F,t}^{\phi-1} \right]^{\frac{\phi}{\phi-1}}, \quad \phi > 0,
\]

where \( a_H \) and \( a_F \) are the weights on the consumption of Home and Foreign traded goods, respectively, and \( \phi \) is the constant (trade) elasticity of substitution between \( C_{H,t} \) and \( C_{F,t} \).
2.1.2 Price indexes

The price index of the Home goods is given by:

\[ P_{H,t} = \left[ \int_0^1 P_t(h)^{1-\phi} dh \right]^{1/\phi}, \quad (4) \]

and the price index associated with the consumption basket, \( C_t \), is:

\[ P_t = \left[ a_H P_{H,t}^{1-\phi} + a_F P_{F,t}^{1-\phi} \right]^{1/\phi}. \quad (5) \]

Let \( E_t \) denote the Home nominal exchange rate, expressed in units of Home currency per unit of Foreign currency. The real exchange rate (RER) is customarily defined as the ratio of CPIs expressed in the same currency, i.e., \( Q_t = \frac{E_t P_t^*}{E_t P_t} \). The terms of trade (TOT) are instead defined as the relative price of domestic imports in terms of exports: \( T_t = \frac{P_{F,t}}{E_t P_t^*} \) if firms set prices in local currency and \( \frac{E_t P_{F,t}^*}{P_{H,t}} \) under producer currency pricing.

2.1.3 Budget constraints

Home and Foreign agents trade an international bond, \( B_{H} \), which pays in units of Home currency and is zero in net supply. Households derive income from working, \( w_t L_t \), from domestic firms’ profits, \( \Pi(h) \), lump-sum transfers \( T_t \), and from interest payments, \( (1+i_t)B_{H,t} \), where \( i_t \) is the nominal bond’s yield, paid at the beginning of period \( t \) but known at time \( t-1 \). Households use their disposable income to consume and invest in state-contingent assets. The individual flow budget constraint for the representative agent \( j \) in the Home country is therefore:

\[ P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + B_{H,t+1} \leq w_t L_t + (1+i_{t-1})B_{H,t} + \int_0^1 \Pi(h) dh + T_t. \quad (6) \]

The household’s problem thus consists of maximizing lifetime utility, defined by (1), subject to the constraint (6).

2.2 Firms

Firms employ domestic labor to produce a differentiated product \( h \) according to the following linear production function:

\[ Y(h) = \zeta_Y L(h), \quad (7) \]

where \( L(h) \) is the demand for labor by the producer of the good \( h \) and \( \zeta_Y \) is a technology shock common to all producers in the Home country, which follows a statistical process to be specified below.

Firms are subject to nominal rigidities à la Calvo so that, at any time \( t \), they keep their price fixed with probability \( \alpha \). We assume that when firms update their prices, they do so simultaneously in the Home and Foreign markets. Following the literature, we consider two models of nominal price distortions in the export markets. According to the first model, firms can set prices in local currencies — this is the LCP hypothesis. The maximization problem is
then as follows:

\[
\max_{\mathcal{P}(h), \mathcal{P}^*(h)} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \mathbb{E}_t \prod_{t+k}^{t+k} \left( \left[ \mathcal{P}_t(h) D_{t+k}(h) + \mathcal{E}_t \mathcal{P}^*_t(h) D^*_t(h) \right] - MC_{t+k}(h) \left[ D_{t+k}(h) + D^*_t(h) \right] \right) \right\} 
\]  

(8)

where \( p_{bt,t+k} \) is the firm’s stochastic nominal discount factor between \( t \) and \( t+k \), and the firm’s demand at Home and abroad is given by:

\[
D_t(h) = \int \left( \frac{\mathcal{P}_t(h)}{\mathcal{P}_H} \right)^{-\theta} C_{H,t} dh \\
D^*_t(h) = \int \left( \frac{\mathcal{P}^*_t(h)}{\mathcal{P}^*_H} \right)^{-\theta} C^*_{H,t} dh 
\]

In these expressions, \( \mathcal{P}_H \) and \( \mathcal{P}^*_H \) denote the price index of Home goods in the Home and Foreign countries — the latter expressed in Foreign currency.

By the first-order condition of the producer’s problem, the optimal price \( \mathcal{P}_t(h) \) in domestic currency charged to domestic customers is:

\[
\mathcal{P}_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}(h) MC_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D_{t+k}(h)} ;
\]  

(9)

while the price (in foreign currency) charged to customers in the Foreign country is:

\[
\mathcal{P}^*_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D^*_t(h) MC_{t+k}(h)}{E_t \sum_{k=0}^{\infty} \alpha^k p_{bt,t+k} D^*_t(h)} .
\]  

(10)

According to the alternative model, we posit that firms set prices in the producer currency — this is the PCP hypothesis. In this case, exchange rate pass-through is complete. Given that demand elasticities are the same across markets, in domestic currency the price charged to foreign consumers is the same as the optimal price charged at Home: the law of one price holds: \( \mathcal{P}^*_t(h) = \mathcal{P}_t(h)/\mathcal{E}_t \). The optimal price is similar to (9), whereas Home demand is replaced by global demand.

Since all the producers that can choose their price set it to the same value, we obtain the following equations for \( \mathcal{P}_H \) and \( \mathcal{P}^*_H \)

\[
P^1_{H,t} = \alpha \mathcal{P}_{H,t-1} + (1 - \alpha) \mathcal{P}_t(h)^{1-\theta} ,
\]  

(11)

\[
P^1_{H,t} = \alpha \mathcal{P}^*_{H,t-1} + (1 - \alpha) \mathcal{P}^*_t(h)^{1-\theta} .
\]

Similar relations hold for the Foreign firms.
2.3 Asset markets and exchange rate determination

In specifying the asset market structure, we restrict trade to one financial instrument only, a safe nominal bond. While capturing the notion that international financial markets do not provide efficient risk insurance against all shocks, intertemporal trade still implies forward-looking exchange rate determination, as a by-product of equilibrium in financial markets. Namely, by combining the Euler equations for the Home households

\[
\frac{U_C(C_t, \zeta_{C,t})}{P_t} = (1 + i_t) E_t \left[ \beta \frac{U_C(C_{t+1}, \zeta_{C,t+1})}{P_{t+1}} \right]
\]

and the Foreign households:

\[
\frac{U_C(C^*_t, \zeta^*_{C,t})}{P^*_t} = (1 + i^*_t) E_t \left[ \beta \frac{U_C(C^*_{t+1}, \zeta^*_{C,t+1})}{P^*_{t+1}} \right],
\]

\[
\frac{U_C(C^*_t, \zeta^*_{C,t})}{E_t P^*_t} = (1 + i_t) E_t \left[ \beta \frac{U_C(C^*_{t+1}, \zeta^*_{C,t+1})}{E_{t+1} P^*_{t+1}} \right]
\]

efficient trade in the international bond will imply the following uncovered interest parity condition, which equates the nominal stochastic discount rates in expectations:

\[
E_t \left[ \beta \frac{U_C(C_{t+1}, \zeta_{C,t+1})}{P_t} \frac{P_t}{P_{t+1}} \right] = E_t \left[ \beta \frac{U_C(C^*_{t+1}, \zeta^*_{C,t+1})}{U_C(C^*_t, \zeta^*_{C,t})} \frac{E_t P^*_t}{E_{t+1} P^*_{t+1}} \right]
\] (12)

Solved forward, this equation pins down the equilibrium exchange rate.

Under complete markets, the condition (12) holds state-by-state, rather than in expectations, since agents trade in contingent assets up to the point when, at the margin, the valuation of an extra unit of currency is equalized across borders. When countries are symmetric, this implies that the relative utility value of wealth, denoted by \(W_t\),

\[
W_t \equiv \frac{U_C(C^*_t, \zeta^*_{C,t})}{U_C(C_t, \zeta_{C,t})} \frac{1}{E_t P_t} = \frac{U_C(C^*_t, \zeta^*_{C,t})}{U_C(C_t, \zeta_{C,t})} \frac{1}{Q_t}
\] (13)

is identically equal to one (see, e.g., Gravelle and Rees [1992], Backus and Smith [1993] and Obstfeld and Rogoff [2001]). Note that the marginal utility of consumption across borders is adjusted for the respective prices of the consumption basket.

Under incomplete markets, however, the equilibrium condition (12) only holds in expectations: any shocks will induce a wedge in the (ex post) relative value of wealth across borders, so that in general \(W_t \neq 1\). As shown below, \(W_t\) defines a theoretically grounded and efficient measure to account for asset markets imperfections in the policy problem—in line with the approach by Woodford [2010], who studies monetary trade-offs under financial frictions in a closed economy setting.

2.4 Log-linearized equilibrium

Throughout the paper, the model’s equilibrium conditions and constraints will be written out in log-deviations from the non-stochastic steady state—we will assume a symmetric steady-state
in which the net foreign asset position is zero and the markup distortion is eliminated, with appropriate subsidies. Details on the log-linearized model equations are given in appendix.

Notation-wise, denoting steady-state values of variable with an upper bar, we will write \( \hat{x}_t = \ln x_t / \mathcal{F} \) for deviations from steady state under sticky prices. While we will study different specifications of the model—PCP vs. LCP, with either unitary or generic trade elasticity—we will not denote variables differently across them, since each specification will be discussed in a separate section or subsection. We make two exceptions to this notation convention. First, we will use the superscript \( fb \) to denote variables in the unique “first-best” allocation, corresponding to the case of complete asset markets, flexible prices and no markup distortions. Second, in Sections 4 and 5, we will use the superscript \( na \) to denote variables in the “natural” (flex-price) allocation when the trade elasticity is set to one or left unconstrained.

Before proceeding, it is useful to single out two properties of the log-linearized equilibrium in our bond economy. First, the uncovered interest parity condition (12) implies

\[
E_t \hat{W}_{t+1} = \hat{W}_t. \tag{14}
\]

Because of incomplete risk sharing, shocks will generally result in a unit root in the relative value of wealth across borders—corresponding to a unit root in net foreign assets. While we will carry out our analysis using a specification of the model in which \( \hat{W}_t \) (and net foreign wealth) is not stationary, in the appendix we show that nonstationarity does not play any substantive role in our result. Second, under a symmetric steady state with zero net foreign wealth, up to first order, net foreign assets (and thus \( \hat{W}_t \)) do not respond to the ex post returns on internationally traded assets. In other words, real net foreign assets are capitalized at the steady-state real interest rate \( \beta^{-1} \). This feature has important implications for optimal monetary policy; namely, starting from a symmetric steady state with zero net foreign wealth, monetary policy cannot correct misallocations in demand and misalignment by manipulating the ex post return on existing assets to affect the wealth distribution (as in, e.g., Devereux and Sutherland [2008] and Benigno [2009]). Instead, it will operate via relative prices, output allocation and net foreign assets accumulation.

3 Monetary policy trade-offs in open economies with incomplete markets

Our main objective is to examine the monetary policy trade-offs brought about by inefficient capital flows in economies where asset markets are incomplete. In this section, we first define and discuss the welfare-relevant gaps shaping policy trade-offs in open economies. We then derive a general quadratic policy loss function obtained from a second-order approximation of agents’ utility for generic incomplete markets (i.e., without specifying the form of market incompleteness). Finally, we characterize the optimal cooperative policy under commitment, in terms of optimal targeting rules. To complete our analysis of monetary policy under incomplete markets, in an appendix we also reconsider how imperfect risk sharing affects the monetary transmission to macroeconomic variables.
3.1 Welfare-relevant gaps in an open economy

As is customary in monetary stabilization analysis, we will write policy objectives and targeting rules in terms of welfare-relevant gaps (all denoted with a tilde), expressing relevant variables as deviations from their first-best allocation values.

3.1.1 The first-best allocation benchmark

Under the assumption that real net foreign assets are zero in steady state ($B = 0$), the first-best allocation can be characterized as follows. The first-best output in the Home and Foreign country, $\tilde{Y}^{fb}_{H,t}$ and $\tilde{Y}^{fb}_{F,t}$, are, respectively:

$$\tilde{Y}^{fb}_{H,t} = \frac{2aH(1-aH)(\sigma \phi - 1)(\tilde{\zeta}_{C,t} - \hat{\zeta}^*_{C,t}) + \hat{\zeta}_{Y,t}(1+\eta)\hat{\zeta}_{Y,t}}{\eta + \sigma}$$

$$\tilde{Y}^{fb}_{F,t} = \frac{2aH(1-aH)(\sigma \phi - 1)(\tilde{\zeta}_{C,t} - \hat{\zeta}^*_{C,t}) + \hat{\zeta}_{C,t}(1+\eta)\hat{\zeta}_{Y,t}}{\eta + \sigma}.$$ (15)

The terms of trade and the real exchange rate are:

$$\tilde{T}^{fb}_{t} = \frac{\sigma(\tilde{Y}^{fb}_{H,t} - \tilde{Y}^{fb}_{F,t}) - (2aH - 1)(\tilde{\zeta}_{C,t} - \hat{\zeta}^*_{C,t})}{4(1-aH) aH (\sigma \phi - 1) + 1}.$$ (16)

$$\tilde{Q}^{fb}_{t} = (2aH - 1) \tilde{T}^{fb}_{t} = \sigma(\tilde{C}^{fb}_{t} - \hat{C}^{*fb}_{t}).$$

The cross-border financial flows, characterized up to first order, are:

$$\hat{B}^{fb}_{t} - \beta^{-1}\hat{B}^{fb}_{t-1} = (1 - aH) \sigma^{-1} \left[(2aH (\sigma \phi - 1) + 1 - \sigma) \tilde{T}^{fb}_{t} - (\hat{\zeta}_{C,t} - \hat{\zeta}^*_{C,t})\right].$$ (17)

where, with slight abuse of notation, $\hat{B}^{fb}_{t}$ refers to “notional” real net foreign assets in the first best, and real net foreign assets, $B_t = \frac{B_{H,t+1}}{P_t}$, are scaled with steady-state output, so that $\hat{B}^{fb}_{t} \approx B^{fb}_{t} - \frac{B}{Y^{fb}}$.

For the purpose of our analysis, the key property of the first-best allocation is that financial and trade flows, as well as relative prices, only respond to shocks affecting contemporaneous (not future, anticipated) productivity and preferences. A notable implication is that neither the short-term real interest rate (given by the growth rates in marginal utility), nor the long-term interest rate (equal to current consumption) moves in response to anticipated shocks.

3.1.2 Misalignment: real exchange rate gaps

A recurrent theme in policy debates concerns the possibility that international relative prices are misaligned and cross-border borrowing/lending is too high or too low—corresponding to either excessive or insufficient demand in different countries. Drawing on previous work of ours (Corsetti et al. [2010]), we now define gaps to account for these policy concerns, using the same logic underlying the definition of the welfare-relevant output gap.

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15 This is so because the model economy abstracts from capital accumulation and other sources of sluggish adjustment, such as habits or adjustment costs. Introducing these features would change the results to follow mainly quantitatively.
Exchange rates are misaligned when they deviate from the value they would take in the efficient allocation.\textsuperscript{16} Since there are different measures of international relative prices, there are different (complementary) measures of misalignment. For the relative price of consumption across countries, the welfare-relevant gap is $\bar{RER}_t$:

$$\bar{RER}_t = \hat{Q}_t - \hat{Q}_t^{fb}. \tag{18}$$

Analogously, for the relative price of tradables, the terms-of-trade gap is $\bar{TOT}_t$:

$$\bar{TOT}_t = \hat{T}_t - \hat{T}_t^{fb}. \tag{19}$$

Finally, misalignments can also occur when nominal rigidities in local currency translates into cross-border deviations from the law of one price (henceforth LOOP). In this case, identical goods are inefficiently traded at different prices at Home and abroad. These price differences define another dimension of misalignment, which, measured on average for the basket of Home goods, is:

$$\bar{\Delta}_H,t = (\hat{E}_t + \hat{P}_t^* - \hat{P}_{H,t}) \tag{20}$$

where $\bar{\Delta}_H,t$ is equal to zero when the LOOP holds. Note that, to the extent that $P_{H,t}$ and $P_{H,t}$ are sticky, the law of one price is violated with any movement in the exchange rate. Specifically, domestic currency depreciation tends to increase the Home firms’ receipts in Home currency from selling goods abroad, relative to the Home market: Home currency depreciation raises $\Delta_H,t$. Similar considerations apply to $\Delta_F,t$.

### 3.1.3 Demand misallocation and the wealth gap

Inefficient external positions could be captured by tracing capital flows in excess of the financial flows in an efficient allocation, i.e., $\bar{B}_t - \bar{B}_t^{fb}$.\textsuperscript{17} However, there is a better, more direct measure of policy-relevant distortions associated with cross-border misallocations. This is the “relative demand gap,” denoted by $\bar{D}_t$, and defined as the cross-country difference in private (consumption) demand relative to the first best:

$$\bar{D}_t = \bar{C}_t - \bar{C}_t^*. \tag{21}$$

One key advantage of $\bar{D}_t$ is that, combined with the real exchange rate gap, $\bar{Q}_t$, it adds up to the “wealth” gap, $\bar{W}_t$, defined as follows:

$$\bar{W}_t = \sigma \bar{D}_t - \bar{Q}_t, \tag{21}$$

\textsuperscript{16}We stress that, conceptually, the first-best exchange rate is not necessarily (and in general will not be) identical to the “equilibrium exchange rate,” traditionally studied by international and public institutions, as a guide to policy-making. “Equilibrium exchange rates” typically refer to some notion of long-term external balance, against which to assess short-run movements in currency values possibly reflecting nominal rigidities and all kinds of real and financial frictions. On the contrary, the efficient exchange rate is theoretically and conceptually defined, at any time horizon, in relation to a hypothetical economy in which all prices are flexible and markets are complete. In fact, our measure of misalignment (as the difference between current exchange rates and the efficient one) is constructed, in strict analogy to the notion of a welfare-relevant output gap, as the difference between current output and the efficient level of output, which does not coincide with the natural rate (i.e., the level of output with flexible prices).

\textsuperscript{17}It is worth stressing that this measure would be well defined also under financial autarky, whereas $\bar{B}_t = 0$. 

13
where $\tilde{W}_t$ is equal to log-deviations in the relative value of wealth (13). If markets are complete, $\tilde{W}_t = 0$ always, even when the overall allocation is not efficient because of nominal rigidities or other distortions. If markets are incomplete, instead, $\tilde{W}_t$ will generally not be zero, and can have either sign, with a straightforward interpretation. A positive gap $\tilde{W}_t > 0$ means that, given the relative price of consumption, the consumption of the Home (national representative) individual is inefficiently high vis-à-vis foreign consumption. While consumption smoothing is optimal from an individual-agent perspective in response to anticipated shocks, from a global welfare perspective relative Home wealth would be too high. Conversely, a negative gap suggests that relative Home demand is inefficiently low given the exchange rate, and/or, for a given $\tilde{D}_t$, the shock causes inefficient real depreciation (relative to first best).

### 3.2 Why and how do incomplete markets affect monetary policy?

The wealth gap defined in the previous subsection fully captures the implications of imperfect financial markets for the policy trade-offs faced by policy-makers in the design of optimal stabilization rules. Under complete markets, $\tilde{W}_t = \sigma \tilde{D}_t - \tilde{Q}_t = 0$. The demand gap $\tilde{D}_t$ and real exchange rate misalignment $\tilde{Q}_t$ can each be different from zero—depending on the effect of nominal rigidities or other distortions (e.g., taxes or markup shocks). Yet, as a consequence of perfect risk sharing, they will always remain proportional to each other: closing $\tilde{Q}_t$ will be tantamount to closing $\tilde{D}_t$, and vice versa. Under incomplete markets, instead, since $\tilde{W}_t$ will generally deviate from zero, $\tilde{D}_t$ and $\tilde{Q}_t$ are no longer proportional to each other. In general, the optimal monetary rule will not close any of these gaps completely, but will have to trade off minimizing these gaps with inflation and output gaps.

In some notable cases (which we analyze in detail in Section 4), capital flows and the corresponding wealth gap $\tilde{W}_t$ will be exogenous to policy: this means that the monetary authorities will not be able to affect the combined inefficiencies arising from both the misallocation in demand and the real exchange rate misalignment.

The wealth gap $\tilde{W}_t$ affects all gaps in the economy. Notably, it impinges on relative price misalignments as follows

$$\tilde{T}_t + \tilde{\Delta}_t = \frac{\sigma (\tilde{Y}_{H,t} - \tilde{Y}_{F,t}) - (2a_H - 1) (\tilde{W}_t + \tilde{\Delta}_t)}{4a_H (1 - a_H) (\sigma \phi - 1) + 1},$$
$$\tilde{Q}_t = (2a_H - 1) (\tilde{T}_t + \tilde{\Delta}_t) + \tilde{\Delta}_t$$

where we used the fact that, under symmetry, $\tilde{\Delta}_{H,t} = \tilde{\Delta}_{F,t} = \tilde{\Delta}_t$ (see Engel [2011]). Note that, while $\tilde{Q}_t$ and $\tilde{T}_t$ are a function of each other, they can move differently in response to shocks because of home bias in preferences and deviations from the law of one price. Taking

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18 With incomplete markets, price movements are not efficient. An appreciation of the real exchange rate associated with a Home consumption boom is a leading example of a pecuniary externality. While fully rational from an individual perspective, agents’s decisions to borrow and lend move international relative prices inefficiently. These are no longer correct indicators of relative scarcity: consumption is higher where the price of the consumption bundle is also higher; see Geanakoplos and Polemarchakis [1986].
the difference in budget constraints, we obtain:

$$\sigma \tilde{D}_t = \sigma \left[ -2 \beta^{-1} \left( \tilde{B}_t - \beta \tilde{B}_{t-1} \right) + \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right] - 2 (1 - a_H) \sigma \tilde{T}_t$$

$$+(1 - a_H) \left[ 4 a_H (1 - a_H) (\sigma \phi - 1) - 1 \right] \sigma^{-1} \left[ (2a_H (\sigma \phi - 1) + 1 - \sigma) \tilde{T}_t^b - \left( \tilde{\zeta}_{C,t} - \tilde{\zeta}_{C,t}^* \right) \right].$$

(23)

Everything else equal, capital inflows ($\tilde{B}_t < 0$) cause the demand gap to turn positive, $\tilde{D}_t > 0$.

The wealth gap affects inflation dynamics both directly and indirectly (via relative price misalignment). The Phillips Curves in our model (four under LCP, collapsing into two under PCP) are written below:

$$\pi_{H,t} - \beta E_t \pi_{H,t+1} = \left( \frac{1 - \alpha \beta}{\alpha} \right) \left[ (\sigma + \eta) \tilde{Y}_{H,t} + \hat{\mu}_t + (1 - a_H) \left[ 2a_H (\sigma \phi - 1) \left( \tilde{T}_t + \Delta_t \right) - \tilde{\Delta}_t - \tilde{W}_t \right] \right]$$

$$\pi_{F,t} - \beta E_t \pi_{F,t+1} = \left( \frac{1 - \alpha \beta}{\alpha} \right) \left[ (\sigma + \eta) \tilde{Y}_{F,t} + \hat{\mu}_t^* + (1 - a_H) \left[ 2a_H (\sigma \phi - 1) \left( \tilde{T}_t + \Delta_t \right) - \tilde{\Delta}_t - \tilde{W}_t \right] \right]$$

where we have also included exogenous markup shocks denoted with $\hat{\mu}_t$ and $\hat{\mu}_t^*$ (see CDL [2010] for a way to extend our setup to model these shocks). These expressions make it apparent that the wealth gap is isomorphic to inefficient (but exogenous) markup shocks. Indeed, when markets are incomplete, the distinction between “efficient” and “inefficient” shocks becomes less useful for the purpose of policy design. Also shocks to tastes and technology endogenously open a wealth gap and create misalignments—and thus raise meaningful policy trade-offs between output and inflation under both LCP and PCP.

In the appendix, we show how imperfect risk sharing—the wealth gap—impinges on the transmission of monetary policy. Notably, we show that, when $\tilde{W}_t$ is exogenous, monetary policy always moves the demand gap and misalignment in the opposite direction, regardless of LCP and PCP. A Home monetary tightening narrows the former while making the real exchange more overvalued.19 Conversely, when capital flows and $\tilde{W}_t$ respond to monetary policy, the effect of a Home monetary tightening depends on structural features such as risk aversion $\sigma$, the trade elasticity $\phi$, the degree of openness and price rigidities. In the appendix we derive threshold values of the trade elasticity (as a function of openness, intertemporal elasticities and the degree of nominal rigidities), above (below) which contractionary monetary policy always increases capital inflows but narrows $\tilde{W}_t$ (vice-versa). These thresholds differ across PCP and LCP economies.

19Yet, as shown in Section 4, monetary policy will still be able to determine in a constrained-efficient way how to spread the welfare costs of macroeconomic adjustment across the different gaps, including the two components of $\tilde{W}_t$. 

15
3.3 A general (quadratic) global policy loss function

From the model, we derive a second-order approximation of the equally weighted sum of the utility of the Home and Foreign national representative agents—written in terms of the gaps defined above, all in quadratic forms. The derivation is presented in detail in the appendix.

The policy loss functions include not only “internal” objectives (inflation and output gaps), but also “external” ones (relative price misalignments and the relative demand gap). Specifically, under the assumption of appropriate subsidies offsetting firms’ markup to deliver an efficient, non-distorted steady state, the period-by-period quadratic welfare function for incomplete market economies is as follows:

\[ L_t^W = (L_t^W)^{fb} \times \frac{1}{2} \left\{ (\sigma + \eta) \left( \tilde{Y}_{H,t}^2 + \tilde{Y}_{F,t}^2 \right) + \frac{\alpha}{(1 - \alpha \beta) (1 - \alpha)} \theta \left( \pi_t^2 + \pi_t^{*2} \right) - \frac{2 \sigma (1 - a_H)}{2a_H (1 - a_H) (\sigma \phi - 1) + 1} \left[ (\sigma \phi - 1) \sigma \left( \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right)^2 - \phi \left( \tilde{\Delta}_t + \tilde{\tilde{W}}_t \right)^2 \right] \right\} + t.i.p., \]

where for convenience we have substituted out terms-of-trade misalignments using their equilibrium relation with output gaps, deviations from the law of one price, and relative demand gaps. While this loss function is written for an LCP economy, its PCP counterpart can be readily obtained by setting the LOOP deviations to zero (\( \Delta_t = 0 \)), and using the fact that, under the law of one price, the inflation term \( \pi_t^2 = a_H \pi_{H,t}^2 + (1 - a_H) \pi_{F,t}^2 \) and \( \pi_t^{*2} = a_H \pi_{H,t}^{*2} + (1 - a_H) \pi_{F,t}^{*2} \) reduces to \( \pi_t^2 = \pi_{H,t}^2 \) and \( \pi_t^{*2} = \pi_{F,t}^{*2} \). It can be shown that expression (24) encompasses the cases of financial autarky (no asset is traded internationally), international trade in one bond, as well as international trade in any number of assets, including complete markets. In this sense, the above function generalizes and complements the ones derived in previous work of ours (CDL [2010]) for the case of autarky and complete markets.\(^{21}\)

3.4 Optimal targeting rules in bond economies

To characterize the optimal cooperative policy under commitment, we maximize the present discounted value of the sum of (24) over time, subject to the log-linearized equilibrium conditions and constraints characterizing the competitive equilibrium allocation in bond economies. In the interest of transparency and tractability, we adopt a timeless perspective (see, e.g., Woodford [2010]), and focus on the (widely studied) case of economies whereas non-contingent bonds are the only assets traded across borders. The derivation is also in the appendix.

Following a standard practice in international economics, the optimal cooperative policy can be synthesized in terms of two targeting rules: a global rule summing up inflation and output gaps across countries, and a cross-country rule, expressed in terms of differences in gaps across

\(^{20}\) Similarly, in related work we show that the loss-function under the case of asymmetric ERPT with DCP, stressed by Gopinath [2016], is a particular case of the above loss-function under symmetric LCP.

\(^{21}\) Gaps (other than output gaps and inflation) similar to the ones we use in our analysis identify policy objectives arising from heterogeneity among sectors and agents in economies distorted by financial imperfections, in addition to nominal rigidities (see, e.g., Cúrdia and Woodford [2016] for an analysis in a closed economy).
countries. From a global perspective, the optimal targeting rule is

\[
0 = \left( \dot{Y}_{H,t} - \ddot{Y}_{H,t-1} \right) + \left( \dot{Y}_{F,t} - \ddot{Y}_{F,t-1} \right) + \theta \left[ a_H \pi_{H,t} + (1 - a_H) \pi_{F,t} + a_H \pi_{F,t}^* + (1 - a_H) \pi_{H,t}^* \right];
\]

where in the case of a PCP economy the inflation term becomes \( \pi_{H,t} + \pi_{F,t}^* \) — as noted above under PCP, world CPI and PPI inflation rates coincide. From a global perspective, the optimal cooperative monetary policy stabilizes output gaps and inflation at the global level. To the extent that world inflation is zero (in the absence of exogenous markup shocks), the sum of output gaps and consumption deviations is also zero. An important implication is that the optimal monetary stance will have the opposite sign across countries. Another implication is that we can write \( \ddot{D}_t \equiv \ddot{C}_t - \ddot{C}_t^* = 2\ddot{C}_t \). These results also hold in the natural rate allocation.

For the cross-country or country-specific rules, tractable general expressions—comparable to the global rule—can be derived only under some parameter restrictions, since deriving these rules involves solving a system of difference equations in the different Lagrange multipliers from the optimal policy problem. We will thus analyze the LCP and PCP economies in turn.

### 3.4.1 Low pass-through (LCP) economies

In the LCP case, a tractable rule is derived by Engel [2011] under the assumptions that markets are complete and \( \eta = 0 \) (infinite labor elasticity). An important result in our paper is that, as long as labor elasticity is infinite, it is possible to derive a tractable cross-country targeting rule also under incomplete markets. This is given by the following expression:

\[
0 = \theta (\pi_t - \pi_t^*) + \ddot{D}_t - \ddot{D}_{t-1} + \frac{4a_H (1 - a_H) \phi (\sigma - 1)}{2a_H (\phi - 1) + 1\sigma} \left[ (\ddot{W}_t - \ddot{W}_{t-1}) + (\ddot{\Delta}_t - \ddot{\Delta}_{t-1}) \right].
\]  

(25)

The targeting rule under complete markets is given by the first two terms on the right hand side of this above expression—such that the cross-country targeting criterion involves only CPI inflation and consumption differentials. The last term, in the wealth gap and deviations from the law of one price, is specific to incomplete markets economies.

The key result derived by Engel [2011] under complete markets is that, as long as \( \eta = 0 \), the relative prices \( \ddot{\pi}_t + \ddot{\Delta}_t \) are exogenous with respect to monetary policy—for any value of \( \sigma \). In the appendix, we are able to establish that the same result also holds under incomplete markets, if we restrict agents to have log-utility, i.e., \( \sigma = 1 \): in LCP economies with \( \eta = 0 \) and \( \sigma = 1 \), monetary policy cannot affect \( \ddot{\pi}_t + \ddot{\Delta}_t \). This result will have notable implications for the analysis in the rest of the paper. Since, as shown further below, cross-border capital flows are solely a function of \( \ddot{\pi}_t + \ddot{\Delta}_t \), it follows that, under LCP, capital flows are independent of monetary policy for any value of the trade elasticity.

Observe that the last term on the right-hand side of the optimal rule (25) drops out when \( \sigma = 1 \): the expression for the cross-country rule (25) is the same under both complete and incomplete markets. However, it does not follow that monetary policy is the same in the two cases. To illustrate the difference, we combine the above expression with the definition of \( \ddot{W}_t \) to rewrite the optimal (cooperative) policy in the form of a country-specific rule for the Home
economy (and a symmetric one for Foreign country). Abstracting from exogenous markup shocks, so that global inflation and global output gaps are both zero under the optimal policy, we can write a country-specific rule as follows:

\[ 0 = \theta \pi_t + \frac{1}{2} \left( (\tilde{W}_t - \tilde{W}_{t-1}) + (\tilde{Q}_t - \tilde{Q}_{t-1}) \right) \]

\[ = \theta \pi_t + \left( \tilde{C}_t - \tilde{C}_{t-1} \right). \]

When markets are complete \((\tilde{W}_t = 0)\), the above reduces to the expression derived by Engel [2011]: with perfect risk insurance, provided that shocks are “efficient” (i.e., they affect tastes and/or technology only, while \(\tilde{\mu}_t = \tilde{\mu}_t^* = 0\)), the optimal policy sets CPI inflation rates to zero. A zero inflation policy closes the consumption gap and eliminates real exchange rate misalignments at once—reflecting the fact that these gaps are proportional to (exogenous) relative prices \(\tilde{T}_t + \tilde{\Delta}_t\). This is not possible when markets are incomplete \((\tilde{W}_t \neq 0)\).

It may be worth stressing that under LCP closing the real exchange rate gap (i.e., setting \(\tilde{e}_Q = 0\)) does not necessarily eliminate deviations from the law of one price—nor prevent inefficient deviations from the law of one price \(\tilde{\Delta}_t\) from mapping into output gap fluctuations. This is apparent from the following expression:

\[ \tilde{Q}_t = (2a_H - 1) \left( \tilde{T}_t + \tilde{\Delta}_t \right) + \tilde{\Delta}_t = (2a_H - 1) \frac{\sigma (\tilde{W}_{F,t} - \tilde{Y}_{F,t}) - (2a_H - 1) \left( \tilde{W}_t + \tilde{\Delta}_t \right)}{4a_H (1 - a_H) (\sigma \phi - 1) + 1} + \tilde{\Delta}_t. \]

Because of nominal distortions in import and export pricing in local currency, the optimal policy allocation cannot be first best, whether or not risk sharing is perfect.

### 3.4.2 High pass-through (PCP) economies

The analytics of the cross-country targeting rule under PCP stands in sharp contrast to the LCP case above. No parameter restriction is required to derive a compact expression for the following cross-country targeting rule in a bond economy:

\[ 0 = E_t \left( \tilde{Y}_{H,t+1} - \tilde{Y}_{H,t} \right) - E_t \left( \tilde{Y}_{F,t+1} - \tilde{Y}_{F,t} \right) + \theta \left( E_t \pi_{H,t+1} - E_t \pi_{F,t+1} \right). \]  \(26\)

Notably, this bond-economy optimal rule is a “forward-looking version” of the cross-country targeting rule under complete markets (see Engel [2011] and CDL [2010]), shown hereafter as:

\[ 0 = \left( \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} \right) - \left( \tilde{Y}_{F,t} - \tilde{Y}_{F,t-1} \right) + \theta \left( \pi_{H,t} - \pi_{F,t}^* \right). \]  \(27\)


Combining once again the global and cross-country rules for bond economies, and abstracting from markup shocks, we can write a country-specific (cooperative) rule for the Home economy:\(22\)

\[ 0 = \left[ \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} + \theta \pi_{H,t} + \frac{2a_H (1-a_H) \phi}{\sigma + \eta (4a_H (1-a_H) (\sigma \phi - 1) + 1)} \right] + \frac{2a_H (\sigma \phi - 1) + 1 - \sigma}{2a_H (\sigma - 1) + 1} \left( \tilde{W}_t - \tilde{W}_{t-1} \right). \]

\(22\) Under financial autarky, the cross-country rule already derived in Corsetti et al. [2010] can be written as...
Note that, if either markets are complete \((\tilde{W}_t = 0)\) or \(\sigma = \phi = 1\), the above expression becomes a function of purely domestic objectives:

\[
\tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} + \theta \pi_{H,t} = 0. \tag{28}
\]

Each country would stabilize its own output gap and GDP-deflator inflation—a result that identifies an important case of “isomorphism” of optimal policy in closed and open economies.

To gain insight on how incomplete markets impinge on key policy trade-offs under PCP, we subtract the two Phillips Curves from each other using the equilibrium expression for the terms of trade:

\[
\pi_{H,t} - \pi_{F,t} = \beta \left( E_t \pi_{H,t+1} - E_t \pi_{F,t+1} \right) + \left( \frac{(1-\alpha \beta) (1-\alpha)}{\alpha} \right) \left\{ \begin{array}{c}
(\eta + \sigma) \left( \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right) + \hat{\mu}_t - \hat{\mu}_t^* \\
-2 (1 - a_H) \cdot \frac{2 a_H (\sigma \phi - 1)}{4 a_H (1 - a_H) (\sigma \phi - 1) + 1} \left( \tilde{Y}_{H,t} - \tilde{Y}_{F,t} \right) - \frac{a_H}{4 a_H (1 - a_H) (\sigma \phi - 1) + 1} \tilde{W}_t \end{array} \right. \right. 
\tag{29}
\]

Under complete markets \((\tilde{W}_t = 0)\), the terms-of-trade gap actually becomes strictly proportional to the differences in output gaps. It is then easy to verify that, consistent with the cross-country targeting rule (28), monetary authorities face no significant trade-off between inflation and output gap stabilization, except in the presence of exogenous markup shocks \(\hat{\mu}_t\) or \(\hat{\mu}_t^*\). In other words, as long as (current and anticipated) shocks affect both output and its first-best counterpart, the optimal policy consists of keeping national output gaps closed and inflation exactly to zero. Under complete markets and PCP, this is indeed the optimal response to (“efficient”) shocks to productivity and preferences.

In the presence of financial imperfections, however, the terms-of-trade gap and output gaps are not proportional to each other. Any shock, including “efficient” shocks to tastes and technology, results in a wealth gap \(\tilde{W}_t \neq 0\), forcing monetary authorities to trade off inflation and output gaps. This is true also when \(\sigma = \phi = 1\), in which case the targeting rules are formally identical across complete and incomplete markets, yet domestic welfare-relevant output gaps do not behave identically.

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follows:

\[
0 = \left[ \sigma + \eta \left( 4 a_H (1 - a_H) (\sigma \phi - 1) + 1 \right) \right] \left\{ \left[ \tilde{Y}_{H,t} - \tilde{Y}_{H,t-1} \right] - \left[ \tilde{Y}_{F,t} - \tilde{Y}_{F,t-1} \right] + \theta \left( \pi_{H,t} - \pi_{F,t} \right) \right\} + \\
4 a_H (1 - a_H) \phi \frac{2 a_H (\sigma \phi - 1) + 1 - \frac{\sigma}{2 a_H (\phi - 1) + 1}}{\tilde{W}_t - \tilde{W}_{t-1}} + \\
2 (1 - a_H) \left[ 2 a_H (\sigma \phi - 1) \sigma - (\sigma - 1) \frac{4 a_H (1 - a_H) (\sigma \phi - 1) + 1}{2 a_H (\phi - 1) + 1} \right] \theta \left( \pi_{H,t} - \pi_{F,t} \right). 
\]
4 Optimal trade-offs varying exchange rate pass-through

In this and the next section, we bring our analysis to bear on the optimal conduct of monetary policy in economies that experience inefficient capital flows and study the macroeconomic dynamics that result from the implementation of the optimal targeting rules. While our analytical results hold in general, throughout our analysis, we will specifically focus on shocks in the form of “news,” indicating anticipated changes in preference parameters. As emphasized by Devereux and Engel [2007], an important reason for analyzing “news shocks” is that they highlight the forward-looking nature of exchange rate determination. From our perspective, a key additional reason is that, as shown in subsection 3.1.1, in the first-best allocation the current values of macro variables do not respond at all to news foreshadowing changes in fundamentals in the future: the response of “gaps” (in anticipation of future changes in technology and preferences) thus coincides with the response in the equilibrium allocation until the anticipated shock materializes—with obvious gains in tractability and analytical transparency.

Throughout the analysis we will posit a linear disutility of labor, $\eta = 0$, as this restriction is necessary for tractability in the LCP economies. Furthermore, for expositional clarity, we will start our analysis by devoting this section to the analysis of a bond economy with a unitary trade elasticity and log-consumption utility—a specification we dub a “Cole and Obstfeld” or CO economy. In the next section, we will generalize our results to the case of non-unitary trade elasticity. The key advantage of beginning our study by assuming a unitary elasticity is that, in this case, preference shocks generate capital flows that are exogenous to policy and macroeconomic adjustments and are thus identical under both LCP and PCP. In a CO economy, therefore, we will be able to compare optimal monetary policy across different economic and policy environments, holding these flows (and thus the underlying shocks) constant.

4.1 A “Cole and Obstfeld” economy with capital flows exogenous to policy

As is well known since Cole and Obstfeld (1991) and subsequent work, in an environment with a Cobb-Douglas aggregator of domestic and imported goods ($\phi = 1$), log consumption utility ($\sigma = 1$) and symmetric home bias, productivity risk is efficiently shared via endogenous terms-of-trade movements, regardless of whether financial markets are complete or incomplete. However, full risk sharing is not granted in the presence of other sources of risk directly affecting net foreign assets, ranging from political risk (i.e., capital controls; see, e.g., Acharya and Bengui [2016]), to shocks to financial intermediation (see, e.g., Gabaix and Maggiori [2015]) and/or preference for foreign assets (see, e.g., Cavallino [2016]), as well as preference shocks impinging on savings. As many of these shocks have broadly similar analytical representations, there is little or no loss of generality in focusing on shocks to preferences that affect the intertemporal valuation of consumption, thus resulting in a motive to save and lend across borders, and generating cross-country capital flows.

Before delving into the analysis of the optimal policy, we highlight a few features of the CO economies that are relevant for our study by contrasting the determinants of capital flows in the first-best allocation and in a bond economy—and by characterizing the natural rate allocation.
4.1.1 Financial flows in the first-best allocation and bond economies

As we have already shown, in the first-best allocation, no macro variable (but the long-term interest rate) responds to news shocks. With \( \sigma = \phi = 1 \), (17) simplifies to

\[
\hat{B}_t^{fb} - \beta^{-1} \hat{B}_{t-1}^{fb} = - (1 - a_H) \left( \hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^* \right).
\]

A surge of (efficient) financial inflows (\( \hat{B}_t^{fb} < 0 \)) can only be driven by contemporaneous relative preference shocks in the Home country (\( \hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^* > 0 \)).

In contrast to the first-best allocation, when the only traded assets are non-contingent bonds, financial flows respond not only to contemporaneous fundamentals, but also to expectations of future fundamentals:

\[
\hat{B}_t = \hat{B}_{t-1} + (1 - a_H) \beta \sum_{j=0}^{\infty} \beta^j E_t \left[ \left( \hat{\zeta}_{C,t+1+j} - \hat{\zeta}_{C,t+1+j}^* \right) - \left( \hat{\zeta}_{C,t+j} - \hat{\zeta}_{C,t+j}^* \right) \right].\tag{30}
\]

An anticipated future fall in the relative degree of impatience (\( \hat{\zeta}_{C,t+1+j} - \hat{\zeta}_{C,t+1+j}^* < 0 \)) causes capital to flow into the Home country—recall that a negative \( \hat{B}_t \) denotes inflows into the Home country. These flows are inefficient: while trade in bonds is welfare maximizing from an individual household’s perspective (at Home and abroad), the entire capital account deficit is excessive relative to the first-best allocation—since in response to anticipated shocks no (notional) capital would flow across borders on impact under perfect risk sharing and flexible prices (i.e., \( \hat{B}_t^{fb} = 0 \)). Note that the size of the inefficient inflows is increasing in openness (decreasing in home bias \( a_H \)). Inefficient capital flows in turn open a wealth gap:

\[
(1 - a_H) \hat{W}_t = - \left( \hat{B}_t - \beta^{-1} \hat{B}_{t-1} \right) - (1 - a_H) \left( \hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^* \right). \tag{31}
\]

Two key results can be derived from the two expressions above. First, in the CO economy, both \( \hat{B}_t \) and \( \hat{W}_t \) are a function of the exogenous preference shocks only, and therefore independent of nominal rigidities and monetary policy regimes. Second, a capital inflow (\( \hat{B}_t < 0 \)) driven by news shocks will invariably lead to a positive wealth gap. From a global welfare perspective, as the Home economy accommodates a higher desire to save among Foreign residents, the relative Home demand \( \hat{D}_t \) grows excessive, and/or the real exchange rate becomes misaligned.

From (30) and (31), it should also be clear that capital inflows are not necessarily associated to a positive wealth gap. Notably, both \( \hat{B}_t \) and \( \hat{W}_t \) can be negative in response to contemporaneous (as opposed to “news”) taste shocks, which raise the utility of current Home consumption (and associated with a relative increase in efficient output, \( \hat{Y}_{H,t}^{fb} > \hat{Y}_{F,t}^{fb} > 0 \)).\footnote{By using (30), you can also write this expression as \( (1 - a_H) E_t \hat{W}_{t+*} = - (1 - a_H) \left[ \beta \sum_{j=0}^{\infty} \beta^j E_t \left[ \left( \hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^* \right) + \left( \hat{\zeta}_{C,t+1+j} - \hat{\zeta}_{C,t+1+j}^* \right) \right] + \frac{1 - \beta}{\beta} \hat{B}_{t-1} \right] \).} In this case, although capital flows into the Home country, domestic consumption is inefficiently low relative to the foreign one. A key difference between contemporaneous and news shocks to preferences is that, with the former, \( \hat{B}_t \) and \( \hat{W}_t \) have the same sign, while with the latter they have the...
opposite sign. The implications of a negative $\tilde{\mathcal{W}}_t$ for optimal monetary policy will be discussed in detail in the next section.

### 4.1.2 The first-best and natural rate allocation

To gain insight into the features of our CO economies, we now rewrite the first-best and the natural-rate allocation. Table 1 shows the first-best allocation (already shown in Section 3) for the case of preference shocks and $\sigma = \phi = 1$, without yet imposing $\eta = 0$ (recall that this assumption is important for tractability under LCP).

<table>
<thead>
<tr>
<th>Table 1. The first-best allocation in the CO economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Y}^{fb}<em>{H,t} = \frac{1}{1 + \eta} \left[ a_H \hat{\zeta}</em>{C,t} + (1 - a_H) \hat{\zeta}^*_C \right]$</td>
</tr>
<tr>
<td>$\hat{Y}^{fb}<em>{F,t} = \frac{1}{1 + \eta} \left[ (1 - a_H) \hat{\zeta}</em>{C,t} + a_H \hat{\zeta}^*_C \right]$</td>
</tr>
<tr>
<td>$\hat{Q}^{fb}<em>{t} = (2a_H - 1) \hat{F}</em>{t}^{fb} = -\frac{\eta}{1 + \eta} (2a_H - 1)^2 \left( \hat{\zeta}_{C,t} - \hat{\zeta}^*_C \right)$</td>
</tr>
<tr>
<td>$\hat{D}^{fb}<em>{t} = \left( \hat{\zeta}</em>{C,t} - \hat{\zeta}^*<em>C \right) - \hat{Q}^{fb}</em>{t}$</td>
</tr>
</tbody>
</table>

If $\eta > 0$, capital inflows from Home preference shocks in favor of current consumption systematically result in a Home currency real appreciation ($\hat{Q}^{fb}_{t} < 0$). But with a linear disutility of labor, $\eta = 0$, the first-best real exchange rate remains constant ($\hat{Q}^{fb}_{t} = 0$). The only effect of the shock is to raise output in both countries, in proportion to the consumption Home bias. Given that the exchange rate is unresponsive, the consumption differential rises efficiently, one-to-one with the contemporaneous relative preference shock: $\hat{D}^{fb}_{t} = \left( \hat{\zeta}_{C,t} - \hat{\zeta}^*_C \right)$.

The flexible price or natural rate allocation for CO economies is shown in Table 2. In the table, all variables are expressed as deviations from the efficient allocations—defining gaps denoted with a superscript “na.” Note the, since $\eta = 0$, $\hat{Q}^{fb}_{t} = 0$, hence $\hat{Q}^{na}_{t} = \hat{Q}_{t}$.

<table>
<thead>
<tr>
<th>Table 2. The natural rate allocation in the CO economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Y}^{na}<em>{H,t} = -\hat{Y}^{na}</em>{F,t} = -1 - a_H \hat{W}_t$</td>
</tr>
<tr>
<td>$\hat{Q}^{na}_{t} = -(2a_H - 1) \hat{W}_t$</td>
</tr>
<tr>
<td>$\hat{T}^{na}_{t} = -\hat{W}_t$</td>
</tr>
<tr>
<td>$\hat{D}^{na}_{t} = 2(1 - a_H) \hat{W}_t$</td>
</tr>
<tr>
<td>$\hat{C}^{na}<em>{t} = -\hat{C}^{na}</em>{t} = \frac{1}{2} \hat{D}^{na}_{t} = (1 - a_H) \hat{W}_t$</td>
</tr>
</tbody>
</table>

With imperfect insurance, inefficient capital flows that open a wealth gap result in misallocation independent of price stickiness. In the natural rate allocation of our CO economies, indeed, output gaps, exchange rate misalignment and the relative demand gap are all proportional to the (exogenous) gap $\hat{W}_t$. When $\hat{W}_t > 0$ and $\hat{B}_t < 0$, as is the case in response to news shocks, capital inflows result in a negative welfare-relevant output gap, an overvalued real exchange rate and an excessive level of domestic consumption, both in absolute terms and relative to Foreigners. Since efficient shocks bring about purely redistributive inefficiencies through their effects on $\hat{W}_t$, the Foreign economy just mirrors the Home responses.

In response to news shocks, as $\hat{W}_t > 0$, all gaps widen on impact. Afterwards, since
$E_t \overline{W}_{t+1} = \overline{W}_t$, gaps remain constant.\(^{24}\) Note that, in the intervening period between the news and future changes in fundamentals, the short-term natural rate of interest (equal to the growth rate of consumption under flexible prices) is not affected at all by the news shocks.\(^{25}\)

It can be shown that, in our CO economies, the natural allocation in Table 2 above not only coincides with PPI price stability under PCP (this is a well known result), but also coincides with CPI price stability under LCP but for relative prices ($\tilde{T}_t + \tilde{\Delta}_t$) and the output gap. This result will be quite useful in the policy analysis to follow.

### 4.2 Domestic demand stabilization with low pass-through (LCP economies)

For given inefficient capital flows (30) and the associated wealth gap (31)), Table 3 shows the constrained-efficient allocation under LCP, when monetary authorities implement the optimal targeting rules, rewritten below for convenience:

$$\theta_{\pi_t} + \frac{1}{2} \left( \tilde{Q}_t - \tilde{Q}_{t-1} \right) = -\frac{1}{2} \left( \tilde{W}_t - \tilde{W}_{t-1} \right),$$

$$2\theta_{\pi_t} = (2a_H - 1) \left( \frac{\beta x_2 - 1}{\beta x_2} \tilde{W}_t - \frac{\beta x_2 - 1}{\beta x_2} \left( \tilde{W}_t - \tilde{W}_{t-1} \right) \right) + (1 - \kappa_1) \tilde{Q}_{t-1}$$

<table>
<thead>
<tr>
<th>Table 3: Constrained-efficient allocation under LCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{Y}_{H,t} = 2a_H (1 - a_H) \left( \tilde{T}_t + \tilde{\Delta}_t \right) + 1/2 \cdot (2a_H - 1) \tilde{D}_t$</td>
</tr>
<tr>
<td>$\theta_{\pi_t} = -(1 - a_H) \left( \frac{\beta x_2 - 1}{\beta x_2} \tilde{W}<em>t + \frac{1}{2} \left[ (\beta x_2 - 1) \tilde{W}</em>{t-1} + (1 - \kappa_1) \tilde{Q}_{t-1} \right] \right)$</td>
</tr>
<tr>
<td>$\tilde{T}<em>t + \tilde{\Delta}<em>t = \nu_1 \left( \tilde{T}</em>{t-1} + \tilde{\Delta}</em>{t-1} \right) - \frac{(\beta y_2 - 1)}{\beta y_2} \tilde{W}_t$</td>
</tr>
<tr>
<td>$\tilde{Q}_t = -(2a_H - 1) \left( \frac{\beta x_2 - 1}{\beta x_2} \tilde{W}<em>t - \frac{1}{\beta x_2} \left( \tilde{W}<em>t - \tilde{W}</em>{t-1} \right) + \kappa_1 \tilde{Q}</em>{t-1} \right)$</td>
</tr>
<tr>
<td>$\tilde{D}<em>t = 2 \left( 1 - a_H \right) \left( \frac{\beta x_2 - 1}{\beta x_2} \tilde{W}<em>t + \frac{1}{\beta x_2} \tilde{W}</em>{t-1} + \kappa_1 \tilde{Q}</em>{t-1} \right)$</td>
</tr>
</tbody>
</table>

In the table, the variables $\kappa_1, \nu_1$ and $\kappa_2, \nu_2$ represent eigenvalues—where $\nu_{1,2}$ differs from $\kappa_{1,2}$ only in that they do not depend on $\theta$.\(^{26}\) For future reference, it is worth noting that

\(^{24}\)When fundamentals change in the future, of course, macroeconomic variables will change again, including both deviations $\tilde{C}^{m}_{t+1}$ and efficient consumption $\tilde{C}^{m}_{t+1}$, but not $\tilde{Q}^{m}_{t}$, under $\eta = 0$.

\(^{25}\)It follows that a monetary policy framework equating the policy rate to the short-term natural rate would be initially unresponsive to the capital inflows.

\(^{26}\)Namely for $\kappa_{1,2}$:

$$\kappa_{1,2} = \frac{1 + \beta + \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \theta \pm \sqrt{\left[ 1 + \beta + \frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha} \theta \right]^2 - 4 \beta}}{2 \beta}$$

and $\nu_{1,2}$ differ from the above only in that the term $\frac{(1 - \alpha \beta)(1 - \alpha)}{\alpha}$ is not multiplied by $\theta$. It is worth noting that the eigenvalues $\kappa_2$ and $\nu_2$ determine the discounted weight attributed to expectations of future fundamentals in driving the dynamics of the real exchange rate and of relative prices $\tilde{T}_t + \tilde{\Delta}_t$. Note that the higher the degree of price stickiness $\alpha$, the larger the stable eigenvalues $\kappa_1$ and $\nu_1$, the lower the speed of adjustment of gaps under the optimal policy. Correspondingly, the lower the unstable eigenvalues $\kappa_2$ and $\nu_2$, the less expected future fundamentals are discounted in determining the gaps.
these eigenvalues are related as follows

\[ 0 < \kappa_1 < 1 < \beta^{-1} + \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha \beta} \theta < \kappa_2, \]

\[ 0 < \nu_1 < 1 < \beta^{-1} + \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha \beta} < \nu_2, \]

\[ \kappa_2 \geq \nu_2 \]

so that \( 0 < \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} < 1, 0 < \frac{(\beta \nu_2 - 1)}{\beta \nu_2} < 1, \) and \( \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \geq \frac{(\beta \nu_2 - 1)}{\beta \nu_2}. \)

Consider the world-economy response to news shocks at time \( t_0, \) resulting in capital inflows and a positive wedge gap \( \hat{W}_{t_0} > 0. \) According to Table 3, implementing the targeting rules leads to a fall in Home CPI inflation on impact, given by the following expression:

\[ \pi_{t_0} = - (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\theta \kappa_2} \hat{W}_{t_0} \leq 0. \quad (32) \]

The (constrained-) optimal contractionary stance at Home does contain the inefficient surge in Home consumption relative to the Foreign one. The relative demand gap is positive

\[ \tilde{D}_{t_0} = 2 (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \hat{W}_{t_0} > 0 \quad (33) \]

but smaller than under CPI price stability (compare with Table 2, whereas \( \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} < 1). \)

The Home real exchange rate correspondingly appreciates on impact:

\[ \tilde{Q}_{t_0} = - \left[ (2a_H - 1) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} + \frac{1}{\beta \kappa_2} \right] \hat{W}_{t_0} < 0. \quad (34) \]

by more than under price stability (since the expression in square brackets is greater than one).\(^ {27} \)

Recall that, since \( \tilde{Q}_{t_0}^{fb} = 0 \) in response to news shocks, the welfare relevant gap and the real exchange rate move one-to-one: \( \tilde{Q}_{t_0} = \hat{Q}_{t_0}. \)

Because the optimal stance is relatively contractionary at Home, the output gap is always smaller than under CPI price stability.\(^ {28} \) Yet, under the optimal policy, the welfare-relevant

\[^{27}\text{Observe that, dynamically, the optimal stance induces a predictable exchange rate dynamic, where Home real appreciation is followed by depreciation. To illustrate this dynamic, one can use the expression for } \tilde{Q}_{t_0} \text{ in Table 3 to decompose the movement of the exchange rate into a long-run permanent appreciation component and a component driven by the expected cumulated real interest rate differential across countries. Comparing the two, what determines this dynamic is the following inequality:} \]

\[ \frac{1}{\beta \kappa_2} > (2a_H - 1) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \frac{\kappa_1}{1 - \kappa_1} = (2a_H - 1) \frac{1}{\beta \kappa_2}. \quad (35) \]

The expected appreciation in the long run reflects the permanent wealth effects associated with the capital inflow under incomplete markets.

\[^{28}\text{Namely:} \]

\[ \hat{Y}_{H,t_0}^{CPI} = - (1 - a_H) \left[ 1 - 2a_H \left( 1 - \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \right) \right] \hat{W}_{t_0}, \]

where now gaps under strict CPI stability are denoted with a CPI superscript.
output gap is not necessarily negative on impact:

\[
\hat{Y}_{H,t_0} = 2a_H (1 - a_H) \left( \hat{T}_{t_0} + \hat{\Delta}_{t_0} \right) + 1/2 \cdot (2a_H - 1) \hat{D}_{t_0} \\
= - (1 - a_H) \left[ \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} - 2a_H \left( \frac{(\beta \kappa_2 - 1)}{\beta \nu_2} - \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \right) \right] \hat{W}_{t_0} \leq 0,
\]

where recall that \( \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \geq \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \). Specifically, it is possible that the positive impact of the capital inflow on the relative demand gap, \( \hat{D}_{t_0} \) outweighs the negative effect of the terms-of-trade gap and deviations from the LOOP, \( \hat{T}_{t_0} + \hat{\Delta}_{t_0} \).\(^{29}\) It is easy to see that, on impact, the output gap is negative if the following condition is satisfied:

\[
\frac{\beta \kappa_2 - 1}{\beta \kappa_2} > 2a_H. 
\]

This condition is more likely to hold in economies that are very open (i.e., economies with a low home bias \( a_H \))—intuitively, openness increases the relative weight of \( \left( \hat{T}_{t_0} + \hat{\Delta}_{t_0} \right) \) and decreases that of \( \hat{D}_{t_0} \) in the output gap expression above. Furthermore, the condition always holds (for any degree of openness), in the limit case where prices are very flexible (\( \kappa_2 \approx \nu_2 \to \infty \)).

Together, these results establish that, in our CO economies under LCP, the monetary authorities optimally trade off stabilization of domestic demand with a larger real exchange rate gap. Using our expressions, we can nonetheless dig deeper, and analyze how this trade-off, i.e., the extent to which monetary policy pursues one objective over the other, varies with the degree of nominal rigidities (thus exchange rate pass-through) and openness.

With respect to the effects of nominal rigidities and pass-through, note that, as prices become flexible at the limit, for \( \alpha \to 0 \), \( \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \to 1 \) and \( \frac{1}{\beta \kappa_2} \to 0 \): in absolute value, the coefficient of \( \hat{W}_{t_0} \) declines in (34), but rises in (33). For higher degrees of price flexibility, optimizing policymakers tolerate a larger misallocation of demand, as they pay more attention to the inefficient real exchange rate appreciation. This is quite intuitive: as import prices become less sticky, exchange rate pass-through is higher. Competitiveness progressively becomes a stronger policy concern relative to aggregate demand stabilization (the more flexible prices are, the closer \( \kappa_2 \) is to \( \nu_2 \), and the smaller the output gap is in absolute value). Remarkably, as prices become less sticky, a milder Home monetary contraction causes the equilibrium rate of inflation (32) to fall by more (since with less nominal rigidities prices react more strongly).

Similar considerations apply to openness: as the economy becomes more open, i.e., for \( a_H \to 1/2 \) (the case of no home bias), the optimal policy leads to a smaller real exchange rate misalignment (since the term \( - (2a_H - 1) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \to 0 \)). In contrast, when the economy becomes more open, a tight domestic monetary policy becomes progressively less effective in dealing with a demand boom fueled by capital inflows, for any given degree of price stickiness.\(^{30}\)

\(^{29}\)Recall that since \( \nu_2 < \kappa_2 \), the expression in square brackets can have either sign.

\(^{30}\)When \( \hat{W}_{t_0} < 0 \)—the case associated with an increase in the efficient level of current output—Home monetary policy is relatively expansionary to stimulate the inefficiently low domestic consumption. Relative to the above, the response of optimal monetary policy is the opposite, because capital inflows are now inefficiently low. The real exchange rate depreciates and is undervalued. However, undervaluation is lower with a high degree of pass-through and openness.
4.3 Exchange rate stabilization and competitiveness with high pass-through (PCP economies)

A comparison of our results across LCP and PCP economies is particularly suitable in our Cole-and-Obstfeld specification, since in response to identical shocks, the sign and size of the ensuing capital flows and wealth gap—that is, the expressions for $b_t$ and $f_{W_t}$ in (31) and (30)—are exactly the same. Conditional on a given $b_t < 0$ and the associated $f_{W_t}$ (positive or negative depending on whether preference shocks are anticipated or contemporaneous), Table 4 presents the allocation under the optimal cooperative monetary policy in the PCP economy.

Table 4: Constrained-efficient allocation under PCP

\[
\begin{align*}
\tilde{Y}_{H,t} &= \kappa_1 \tilde{Y}_{H,t-1} - (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \tilde{W}_t \\
\theta \pi_{H,t} &= (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \tilde{W}_t + (1 - \kappa_1) \tilde{Y}_{H,t-1} \\
\tilde{T}_t &= - \left(1 - 2 \frac{(1 - a_H)}{\beta \kappa_2}\right) \tilde{W}_t + 2 \kappa_1 \tilde{Y}_{H,t-1} \\
\tilde{Q}_t &= - (2a_H - 1) \left[\left(1 - 2 \frac{(1 - a_H)}{\beta \kappa_2}\right) \tilde{W}_t - 2 \kappa_1 \tilde{Y}_{H,t-1}\right] \\
\tilde{D}_t &= 2 (1 - a_H) \left[1 + \frac{(2a_H - 1)}{\beta \kappa_2}\right] \tilde{W}_t + 2 (2a_H - 1) \kappa_1 \tilde{Y}_{H,t-1}
\end{align*}
\]

The Home optimal monetary response to the capital inflows is the opposite relative to the LCP case—since the wealth gap enters the expression for PPI inflation with the opposite sign. To wit, consider again news shocks that cause $b_t < 0$ and $f_{W_t} > 0$. When exchange rate pass-through is complete, Home monetary authorities would implement a monetary expansion. Compared with the natural rate allocation in Table 2, on impact they tolerate some short-run (GDP deflator) inflation:

\[
\pi_t = (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\theta \beta \kappa_2} \tilde{W}_t > 0
\]

and lean on the appreciation of the real exchange rate

\[
\tilde{Q}_t = - (2a_H - 1) \left(1 - 2 \frac{(1 - a_H)}{\beta \kappa_2}\right) \tilde{W}_t < 0.
\]

so as to contain competitiveness losses. Relative to the allocation in Table 2, the expansionary stance mitigates the negative output gap

\[
\tilde{Y}_{H,t_0} = - (1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \tilde{W}_t < 0,
\]

(this is so because $\frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} < 1$), at the cost of increasing the relative demand gap

\[
\tilde{D}_t = 2 (1 - a_H) \left[1 + \frac{(2a_H - 1)}{\beta \kappa_2}\right] \tilde{W}_t > 0.
\]

The optimal degree of monetary expansion again depends on whether the economy is more or less open, and the degree of price stickiness.
4.4 Exchange rate volatility, inflation and output gaps in CO economies: LCP vs PCP

For our CO economies, the macroeconomic response to shocks under the optimal policy is illustrated by Figure 1. The figure plots the impulse responses of the relevant gaps to a preference shock anticipated to occur 20 quarters in the future (outside the time scale of the graph), causing an inflow of capital in the Home economy.\footnote{The parameter values are as follows: \( \eta = 0, \phi = \sigma = 1, a_H = .75, \beta = .99, \alpha = .75, \theta = 3. \)} The shock is normalized to produce an initial capital inflow as high as 1 percent of Home GDP. As shown by the first graph in the upper left corner, the stock of foreign debt increases exogenously along the optimal adjustment path. The size of capital flows is excessive: the wealth gap (shown in the graph in the upper right corner) jumps to a positive value and remains constant, according to (14). Both the capital inflows and the wealth gap are exogenous to macroeconomic adjustment and policy and, hence independent of LCP and PCP.

The remaining graphs in the figure distinguish between LCP economies (continuous lines) and PCP economies (dashed lines). The price response (lower left corner) shows that the monetary stance is relatively expansionary under PCP (GDP-deflator inflation is positive), contractionary under LCP (CPI inflation is negative).

Comparing the two economies highlights an important result. Under the optimal policy, the real exchange rate is always less volatile under PCP (where monetary authorities lean against appreciation) than under LCP (where monetary authorities exacerbate misalignment). Analytically, this follows from observing that under strict inflation targeting, the real exchange rate response under LCP (CPI targeting) is the same as under PCP (GDP deflator targeting), and thus equal to the natural rate allocation \( \hat{Q}_t^{na} = - (2a_H - 1) \hat{W}_t \). Relative to this natural rate allocation, we have shown that the optimal policy makes the real exchange rate less volatile under PCP, and more volatile under LCP. Correspondingly, the real exchange rate always undershoots its long-run value under PCP—and overshoots under LCP. Because of the expenditure-switching effects of the exchange rate, however, the output gap is more negative under PCP.\footnote{Analytically, this follows from comparing the expression for the output gaps under PCP, the natural allocation and LCP, whereas, since \( \nu_2 < \kappa_2 \),

\[
(1 - a_H) \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} > (1 - a_H) \left[ 2a_H \left( \frac{(\beta \nu_2 - 1)}{\beta \nu_2} - \frac{(\beta \kappa_2 - 1)}{\beta \kappa_2} \right) > (1 - a_H) \left[ 1 - 2a_H \left( \frac{(\beta \nu_2 - 1)}{\beta \nu_2} \right) \right].
\]}

As already pointed out in Section 3, in the Phillips curves (29), the wealth gap is ‘isomorphic’ to exogenous markup shocks. To conclude the analysis, it is instructive to stress that, despite their isomorphism, wealth gaps and markup shocks have quite different implications for optimal monetary policy. We have seen above that, in response to an appreciation following capital inflows, the Home monetary response is contractionary under LCP and expansionary under PCP. In contrast, when an appreciation follows an inflationary markup shock, the optimal monetary response is always expansionary, irrespective of LCP and PCP (see, e.g., Engel [2011] or CDL [2010]).

Intuitively, consider an inflationary markup shock \( \mu_{t_0} > 0 \) in the Home country. Under LCP, if monetary policy is either conducted optimally, or pursues CPI price stability, an inflationary markup shock appreciates the Home currency in real terms. Differently from the case of capital
inflows associated with $\tilde{W}_t > 0$, however, an exogenous markup shock only creates a trade-off between inflation and misalignment—without affecting $\tilde{D}_t$. Hence, in order to lean against real overvaluation, monetary policy has to ease to move CPI inflation in the opposite direction.

5 Optimal trade-offs varying trade elasticities

In this section, we generalize our analysis by relaxing the assumption of a unitary trade elasticity. This allows us to extend the analysis in at least three directions. First, unlike in the CO economies, cross-border flows also respond to shocks to productivity—in addition to shocks to preferences for saving and/or changes in taxes or capital controls. We can thus consider different types of business cycle disturbances. Second, capital flows may no longer be exogenous to monetary policy. We can thus characterize how optimal monetary policy affects the size of inefficient cross-border borrowing and lending. Finally, and most crucially, the wealth gap $\tilde{W}_t$ associated with capital inflows (excessive relative to the first-best allocation) can be negative also in response to news shocks. To keep the analytical complexity at a minimum, in the rest of this section we will restrict our attention to “news shocks” only—no contemporaneous shocks will appear in the equations to follow.\footnote{Contemporaneous shocks mainly affect the relation between capital flows and the sign of the wealth gap; nevertheless, given the latter, the optimal monetary policy response is the same for both contemporaneous and anticipated shocks.}

A remarkable conclusion from our analysis is that most of the insight from our CO economies will go through, unaffected, in economies with trade elasticities sufficiently bounded away from zero. Where optimal monetary policy differs—for economies in which tradables are strong complements—the difference rests on key features of the international transmission, best understood in light of the classical controversy on the transfer problem.

5.1 Wealth gaps and monetary policy: insight from the transfer problem

To gain insight on why and how the sign of $\tilde{W}_t$ impinges on monetary policy, it is useful to reconsider, if only briefly, how capital inflows may affect relative wealth and the real exchange rate. We do so drawing on the “transfer problem,” the classical controversy in open economy originated by the debate between Keynes and Ohlin about the effects of war reparation payments on the terms of trade of a country (see Keynes [1929] and Ohlin [1929]). Under incomplete markets, capital inflows into Home are effectively a transfer of income and purchasing power from Foreign, reflecting higher savings by Foreign residents and higher dissaving by Home residents. From a global perspective, if relative prices did not adjust, because of home bias in demand, the transfer would translate into an excess supply of Foreign goods. Equilibrium unavoidably requires a relative price adjustment—as John Williamson would put it, there is no “immaculate transfer” (see Krugman [2007]).

In economies in which, in equilibrium, substitution effects from the real exchange rate are stronger than income effects, adjustment to a transfer occurs via Home real appreciation, redirecting world demand towards Foreign goods. Because of the fall in the relative price of Foreign output, Foreign income falls and Home income rises by more than the size of the transfer at constant prices—the problem stressed by Keynes. The appreciation compounds the rise in Home
relative wealth from the transfer, so that $\tilde{\mathcal{W}}_t > 0$. We will show that our results on the optimal policy derived for the CO economies nicely extend to this case.

The equilibrium adjustment is however quite different if income effects from relative price adjustment are stronger than substitution effects—corresponding to low-elasticity economies in which Home and Foreign goods are strong complements. In response to Home capital inflows there is no equilibrium with Home appreciation/Foreign depreciation, because this would drive Foreign demand too low for the goods markets to clear at global level (see, e.g., CDL [2008a]). Instead, equilibrium requires Foreign appreciation/Home depreciation, with the effect of reducing Home relative wealth—so that $\tilde{\mathcal{W}}_t < 0$—in spite of the transfer.

Stronger income effects relative to substitution effects have a key implication for monetary policy design. As we will show below, in economies where the trade elasticity is sufficiently low that $\hat{B}_t < 0$ and $\tilde{\mathcal{W}}_t < 0$, sustaining domestic demand and output in response to capital inflows and currency undervaluation becomes the overriding concern of monetary policy: the optimal monetary stance is expansionary—for any degree of exchange rate pass-through and openness.\footnote{It is worth stressing that no such effect would materialize were markets complete: perfect risk diversification would eliminate any adverse income effects from shocks and exchange rate movements. Recall from Section 3.3.1 that in the first-best allocation news shocks would not trigger any financial flow across borders, so that in a bond economy all capital inflows in the natural allocation are invariably excessive, irrespective of the sign of $\mathcal{W}_t^{na}$.}

### 5.2 The natural rate allocation with news shocks

As in the previous section, it is convenient to start out with a generalization of the flexible price, natural-rate, allocation for any trade elasticity, shown in Table 2. An important result is that, as long as $\sigma = 1$, $\eta = 0$, the natural rate allocation with $\phi \neq 1$ for the case of news shocks differs from Table 2 only in the following:

$$\tilde{Y}_{H,t}^{na} = -(1 - a_H) [2a_H (\phi - 1) + 1] \tilde{\mathcal{W}}_t^{na} = \tilde{B}_t^{na}. \quad (36)$$

In other words, while news shocks invariably lead to capital inflows ($\hat{B}_t^{na} < 0$) and a negative output gap, the associated wealth gap, $\tilde{\mathcal{W}}_t^{na}$, may be positive or negative, depending on the value of the trade elasticity. Specifically, given $\hat{B}_t^{na} < 0$,

$$\tilde{\mathcal{W}}_t^{na} > 0 \quad \text{if} \quad \phi > \frac{2a_H - 1}{2a_H} \leq 1/2. \quad (37)$$

Once the sign and size of $\tilde{\mathcal{W}}_t^{na}$ is determined, however, all the other gaps behave exactly the same as in Table 2—they therefore depend on the elasticity $\phi$ through the response of the wealth gap. As discussed above, for $\tilde{\mathcal{W}}_t^{na} > 0$, capital inflows appreciate the exchange rate, the Home currency is overvalued and Home domestic demand is excessive. The opposite is true when, for elasticities below the threshold above, $\tilde{\mathcal{W}}_t^{na} < 0$: capital inflows are associated with real depreciation and the Home real exchange rate is undervalued; Home demand is too low.

### 5.3 Low pass-through (LCP) economies

The equilibrium relation between capital flows and the wealth gap in LCP economies is shown in Table 5, together with the full solution for the dynamics of capital flows under the optimal policy. The two expressions in the table depend only on exogenous shocks, and (through the
t.i.p. term)\textsuperscript{35} on the current and anticipated future evolution of relative prices in the first-best allocation, unaffected by policy. This observation establishes an important result, so far unnoticed in the literature. As long as \( \eta = 0 \) and \( \sigma = 1 \), cross-border capital flows and the wealth gap in LCP economies remain independent of policy even if the trade elasticity is different from unity (the case of CO economies).

Table 5: Capital flows under LCP and with news shocks, for \( \phi \neq 1 \)

\[
(1 - a_H) \left[ 1 + 2a_H (\phi - 1) \frac{\beta (\nu - 1)}{\beta \nu} \right] \hat{W}_t = - \left( \hat{B}_t - \beta^{-1} \hat{B}_{t-1} \right) + \\
2a_H (1 - a_H) (\phi - 1) \sum_{j=0}^{\infty} \nu_j (1 - \beta) E_t \left( \left[ \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right] - \beta^{-1} \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) \right)
\]

\[
\hat{B}_t - \hat{B}_{t-1} = \frac{2a_H (\phi - 1) \beta \nu \left( \frac{\beta (\nu - 1)}{\beta \nu} \right) }{1 + 2a_H (\phi - 1) \frac{\beta (\nu - 1)}{\beta \nu}} \beta \sum_{j=0}^{\infty} \nu_j \left( \left[ \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right] - \beta^{-1} \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) \right) + t.i.p
\]

From the table, it is apparent that the trade elasticity \( \phi \) nonetheless matters for \( \hat{B}_t \) and \( \hat{W}_t \) in two crucial respects. First, it determines whether a given “news shock” translates into inefficient borrowing or lending; second, it determines whether \( \hat{B}_t \) and \( \hat{W}_t \) have the same or the opposite sign. Differently from the natural allocation (generally unfeasible in LCP economies), the threshold value of the trade elasticity below which \( \hat{B}_t \) and \( \hat{W}_t \) have the same sign depends on more than the home bias parameter, and is not unique, but conditional on which shocks hit the economy. For the case of taste shocks, the threshold is:\textsuperscript{36}

\[
\phi < \frac{2a_H - \frac{\beta \nu \left( \frac{\beta (\nu - 1)}{\beta \nu} \right)}{2a_H}}{2a_H} < 1.
\]

This threshold is smaller, the more open the economies (\( a_H \rightarrow 1/2, \phi \geq 0 \)) and the higher the degree of price stickiness (\( \nu_2 \rightarrow 1/\beta \), so that \( \frac{\beta \nu \left( \frac{\beta (\nu - 1)}{\beta \nu} \right)}{2a_H} \rightarrow 1, \phi \geq 0 \))—resulting in a lower degree of

\textsuperscript{35}The terms independent of policy (t.i.p.) in the table are:

\[
t.i.p. = \frac{\left[ 1 + 2a_H (\phi - 1) \frac{\beta (\nu - 1)}{\beta \nu} \right] \beta \sum_{j=0}^{\infty} \beta \nu_j \left[ \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) - \beta^{-1} \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) \right]}{1 + 2a_H (\phi - 1) \frac{\beta (\nu - 1)}{\beta \nu}} \beta \sum_{j=0}^{\infty} \nu_j \left( \left[ \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right] - \beta^{-1} \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) \right) +
\]

\[
\sum_{j=0}^{\infty} \beta \nu_j \left[ \left[ \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right] - \beta^{-1} \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) \right] +
\]

\textsuperscript{36}As shown above, with \( \sigma = 1 \) and \( \eta = 0 \), the terms-of-trade response to (current or anticipated) taste shocks in the first-best allocation is \( \hat{T}_t^{fb} = 0 \). So, the expressions in Table 5 simplify as follows:

\[
(1 - a_H) \left[ 1 + 2a_H (\phi - 1) \frac{\beta (\nu - 1)}{\beta \nu} \right] \hat{W}_t = - \left( \hat{B}_t - \beta^{-1} \hat{B}_{t-1} \right) +
\]

\[
\hat{B}_t - \hat{B}_{t-1} = \frac{2a_H (\phi - 1) \beta \nu \left( \frac{\beta (\nu - 1)}{\beta \nu} \right) }{1 + 2a_H (\phi - 1) \frac{\beta (\nu - 1)}{\beta \nu}} \beta \sum_{j=0}^{\infty} \nu_j \left[ \left[ \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right] - \beta^{-1} \left( \hat{T}_{t+j}^{fb} - \hat{T}_{t+j}^{fn} \right) \right] +
\]

from which it is easy to derive the threshold in the text. Note that the first-terms of trade \( \hat{T}^{fb}_{t+j} \) in Table 5 are different from zero for productivity shocks.
pass-through. For the case of anticipated productivity shocks, the threshold is:

$$\phi < \frac{2a_H - \frac{\beta \nu_2}{(\beta \nu_2 - 1)} (1 - \beta \nu_1)}{2a_H} < 1.$$ 

Since $\nu_1 \leq 1 < \beta^{-1} \leq \nu_2$, this expression is unambiguously above the one derived for the case of taste shocks.\(^{37}\)

Remarkably, given the sign and paths of $\tilde{B}_t$ and $\tilde{W}_t$ in response to shocks (from Table 5), the trade elasticity $\phi$ does not enter directly the expressions for the response of inflation, demand gaps and the real exchange rate. Indeed, provided contemporaneous shocks are excluded from the analysis, these expressions are exactly the same as in Table 3, derived for the CO economies with a unitary elasticity. From Table 3, we know that, in response to shocks that cause a capital inflow, $\tilde{B}_t < 0$ associated with a positive wealth gap, $\tilde{W}_t > 0$, the Home monetary authorities will optimally let inflation decline, at the cost of exacerbating the Home real exchange rate appreciation (and overshooting) in the short run—they will implement a monetary tightening.

In line with our earlier analysis, the extent to which the optimal policy response translates into a fall in relative consumption will depend on the degree of openness and stickiness of import prices, the latter in turn determining the degree of exchange rate pass-through.

A variable for which the trade elasticity parameter $\phi$ makes a difference, however, is the welfare-relevant output gap. Its impact response to the optimal contractionary stance is:

$$\tilde{Y}_{H,t_0} = - (1 - a_H) \left[ (\beta x_2 - 1) \beta x_2 - 2a_H \left( \frac{(\beta x_2 - 1)}{\beta x_2} - \frac{\phi (\beta \nu_2 - 1)}{\beta \nu_2} \right) \right] \tilde{W}_{t_0} \leq 0. \quad (38)$$

It is easy to show that the response of this gap to the optimal monetary contraction is unambiguously negative for values of $\phi$ sufficiently above 1.

As mentioned above, the optimal policy response to excessive inflows is quite different in economies where domestic and foreign goods are highly complementary and the effects of the “transfer” change sign, for values of the trade elasticity below the relevant threshold (which, as discussed above, depends on the type of shocks). With $\tilde{B}_t < 0$ and $\tilde{W}_t < 0$, the (exogenous) capital inflows are associated with inefficiently low domestic demand: monetary authorities optimally focus on domestic demand stabilization (see Table 3). The optimal stance is relatively expansionary (rather than contractionary) at Home, up to the point of bringing the Home output gap into positive territory (as follows from assessing (38) for $\phi \to 0$). Relative to strict inflation targeting, Home aggregate demand and economic activity will be stronger, while the real exchange rate will clearly be weaker—i.e., even more undervalued.\(^{38}\)

\(^{37}\)This result is apparent from the fact that $\tilde{B}_t < 0$ necessarily implies that the sum of the last two lines in the second expression in the Table 3 have the opposite sign and are larger in absolute value than the third line in the same expression, which also appears in the equation for $\tilde{W}_t$. The threshold is sufficient for the term $\frac{1}{1 + 2a_H (\phi - 1) \frac{a_H + 1}{a_H + 2}}$ to be non negative.

\(^{38}\)With $\sigma \neq 1$, capital flows and wealth gaps respond to monetary policy. Yet, under reasonably general conditions, the results discussed in this subsection will go through: the sign of monetary policy is not determined by capital flow stabilization. Moreover, as shown in the appendix, for $\sigma > 1$ and $\phi > 1$, expansionary monetary policy always reduces the capital inflow; the opposite happens for $\sigma < 1$ and $0 \leq \phi < 1 - \frac{2(\nu_2 - 1)}{2a_H + 1} < 1$. 

\[31\]
5.4 High pass-through (PCP) economies

The allocation under the optimal policy in PCP economies is shown in Table 6, once again abstracting from contemporaneous shocks. Different from our results under LPC, capital flows are no longer independent of the macroeconomic allocation and therefore of policy. The optimal monetary stance affects the size of the inflows even for $\sigma = 1$.

Table 6: Constrained-efficient allocation under PCP with news shocks, for $\phi \neq 1$

\[
\tilde{W}_t = A \cdot \beta \sum_{j=0}^{\infty} \beta^j \left[ 2aH (\phi - 1) E_t \left( \left( \hat{H}_{B,t+j+1}^* - \hat{H}_{F,t+j+1}^* - \hat{H}_{F,t+j+1}^* \right) \right) + \right. \\
- \left( 2aH (\phi - 1) + 1 \right) E_t \left( \left( \tilde{h}_{C,t+j+1} - \tilde{h}_{C,t+j}^* \right) \right) \left( \tilde{h}_{C,t+j}^* - \tilde{h}_{C,t+1+j} \right) \right] + \\
2 \left( 1 - aH \right) \left[ 2aH (\phi - 1) \frac{1}{4aH (1-aH)(\phi-1)+1} \right] \frac{(1-\beta_1)}{1-\beta_1} \beta \tilde{Y}_{H,t-1} \left( 1 - aH \right) \left[ \frac{2aH (\phi - 1) + 1}{\beta_2} \tilde{W}_t \right] + \\
\frac{2aH (\phi - 1)}{2aH (1-aH)(\phi-1)+1} \tilde{W}_t \left( \beta \tilde{Y}_{H,t-1} \right) \left( 1 - aH \right) \left( \phi - 1 \right) + 1
\]

Three results are worth stressing. First, if exchange rate pass-through is complete, the threshold for the trade elasticity at which the wealth gap $\tilde{W}_t$ switches sign under the optimal policy is the same as the one derived for the natural rate allocation (37), and thus identical for both (anticipated) taste and productivity shocks. Together with the results from the previous subsection, this establishes that the elasticity threshold below which a capital inflow causes a negative wealth gap is never larger (whether under LCP or PCP) than in the natural rate allocation. It is bounded above by $\frac{2aH-1}{2aH}$, which is decreasing in openness (and goes to zero for $a_H \rightarrow 1/2$, the case of no home bias in consumption).

Second, under the optimal policy, the impact response of inflation to capital inflows is always non-positive for any value of $\phi$, namely:

\[
B = \frac{\left( 1 - a_H \right)}{2aH (1-aH)(\phi-1)+1} \left[ 1 - \frac{4aH (1-aH)(\phi-1)+1}{4aH (1-aH)(\phi-1)+1} \frac{4aH (1-aH)(\phi-1)+1}{4aH (1-aH)(\phi-1)+1} \frac{4aH (1-aH)(\phi-1)+1}{4aH (1-aH)(\phi-1)+1} \right] \leq 0.
\]

Second, the sign of the coefficient $A$ multiplying the same term in the expression for $\tilde{W}_t$, given by

\[
A = \frac{\left( 2aH (\phi - 1) + 1 \right)^{-1}}{4aH (1-aH)(\phi-1) + 1 + 4aH (1-aH) \phi \frac{4aH (1-aH)(\phi-1)+1}{2aH (1-aH)(\phi-1)+1} \beta_2 (1-\beta_1)} \leq 0
\]

depends instead on whether $\phi$ is above or below the threshold (37).
positive, for any value of the elasticity $\phi$, i.e., whether $\bar{\mathcal{W}}_t$ is positive or negative:

$$\theta_{\pi_{H,t}} = (1 - a_H) \frac{\beta \sigma_2 - 1}{\beta \sigma_2} \left[ 4a_H (1 - a_H) (\phi - 1) + 1 \right] \frac{\bar{\mathcal{W}}_t}{2a_H (\phi - 1) + 1} > 0.$$ 

Hence, the optimal monetary policy is always expansionary on impact (unlike the case of LCP, there is no switch in the sign of the monetary policy). The trade elasticity, however, impinges on the inflationary impact of the optimal monetary expansion.

For elasticities above the threshold (37), the optimal policy is similar to the one derived in the CO economy. Capital inflows associated with inefficiently high Home demand ($\bar{\mathcal{W}}_t > 0$) call for easier monetary policy at Home (see the inflation expression above), to resist exchange rate overvaluation. The welfare-relevant output gap nonetheless remains negative, despite the fact that such a stance stokes inflationary pressures.

For elasticities below the threshold (37) such that $\bar{B}_t$ and $\bar{\mathcal{W}}_t$ are both negative, the expansionary Home policy stance may even bring the welfare-relevant output gap into positive territory. As shown in the appendix, for low enough trade elasticities, the optimal Home monetary policy stance is increasingly driven by the need to prop up an inefficiently low domestic demand. In response to the capital inflow, Home residents’ consumption actually falls in relative terms for values of $\phi$ below the following threshold:

$$\phi \leq \frac{(2a_H - 1) (\beta \sigma_2 - 1)}{2a_H \beta \sigma_2} \leq \frac{(2a_H - 1)}{2a_H}.$$ 

It is for this region of elasticities that the optimal monetary boost turns the output gap positive.

Our third result is that, unlike the cases studied so far, the monetary expansion now affects the size of capital inflows. In general, there are two channels to consider, working in opposite directions. By leaning against real appreciation, an expansionary monetary policy discourages capital inflows; by sustaining domestic demand, it raises domestic borrowing. As shown in the appendix, under the optimal policy the first channel prevails for $\phi > 1$: relative to the benchmark of the natural rate allocation, capital inflows are smaller in absolute value. The second channel prevails for elasticity values in the range $1 > \phi > \frac{2a_H - 1}{2a_H}$: the optimal monetary stance magnifies net borrowing relative to the natural rate allocation.

6 Conclusions

As a consequence of the 2008 global financial crisis, much research has been devoted to reconsider the set of policy tools and measures that can be activated to insulate national economies from the ebb and flows of cross-border capital flows. In this paper, we have taken the perspective of monetary policy decision making, and analyzed what monetary instruments can deliver when additional tools are not readily available and/or are of limited effectiveness. Our main question is how monetary policy could optimally respond to inefficient capital flows on domestic macroeconomic dynamic and welfare, by optimally trading off domestic and external objectives.

Our study provides key analytical insight into the efficient resolution of this trade-off. When

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41To see why, note that, conditional on $\bar{B}_t < 0$, the term on the right-hand side of the expression in the text is always positive ($2a_H (\phi - 1) + 1)^{-1} \bar{W}_t$ always has the opposite sign of $\bar{B}_t$).

42As shown in the appendix, an expansionary monetary shock always decreases capital inflows for $\phi > 1$. 
international capital markets are imperfect (so that capital flows are associated with currency misalignment), the design of optimal monetary rules hinges on recognizing the direct and indirect relevance of exchange rates for domestic stabilization and welfare. With a high pass-through, indeed, the optimal response to inefficient capital inflows is directed to contain misalignment, tolerating a temporary surge in domestic inflation in case of real overappreciation, or an inflation decline in case of currency undervaluation. Conversely, the stabilization of aggregate demand is a priority—at the costs of missing out on external objectives and temporary below (above) target inflation—when imperfect pass-through mutes the price competitiveness effects of exchange rate overvaluation (undervaluation) —provided trade elasticities are not too low.

It is worth emphasizing that in our study we have derived all these results analytically—enhancing clarity and transparency, and possibly pointing to avenues for generalization and extension. By way of example, our results, derived under commitment, can be brought to bear on the case of cooperation under discretion, where policymakers are not able to improve the short-run trade-offs among competing goals by credibly guiding expectations of future policy rates and inflation. As is well known, in the closed economy counterpart of our model, or in its version under complete markets, optimal targeting rules derived under discretion will include all variables (a part of inflation) in levels, rather than in growth rates. Namely, under discretion monetary authorities cannot credibly pursue a nominal anchor in level. This will also be the case in the specification of our bond economy where capital flows are exogenous to monetary policy (see Sections 4 and 5.3), implying that the wealth gap is not an argument in the optimal targeting rules. In these economies, then, the targeting rules under discretion can be easily derived from our analysis in Section 3—crossing out lagged terms. Economic dynamics can be readily derived from our analysis in Sections 4 and 5. In more general specifications of the model, however, the accumulation of net foreign assets and liabilities will change the state of the economy over time: targeting rules derived under discretion will generally include a term capturing the optimal policy response to foreign debt accumulation, complicating their analytical characterization. By the same token, the analytical structure of the model can be brought to bear on the interactions of monetary policy and capital controls/macro-prudential instruments. These policy tools are intended to address more directly the distortions due to financial frictions, but could also have significant consequences for conventional stabilization policy.

Among the directions for future research, an important one concerns the analysis of strategic interactions among policymakers. Numerical analyses of the Nash equilibrium under incomplete markets and PCP suggest that, although policymakers have an incentive to manipulate the terms of trade of the country in their own national interests, incomplete markets increase the weight attached to stabilization of domestic incomes (see e.g. Rabitsch [2012] and, for a small open economy, De Paoli [2009]). Based on our results, we can further observe that inefficient capital flows have strong redistributive effects across borders. We have seen that cooperative policies attempt to redress these effects: in our analysis, when the optimal monetary policy at Home is either a contraction or an expansion, the Foreign monetary stance has the opposite sign. Without cooperation, however, these redistributive effects of capital inflows inherently create room for conflicts and strategic behavior.

Finally, while in this paper we focus on the benchmark cases of PCP and LCP, the evidence
on the importance of pricing in vehicle (or dominant) currencies strongly motivates further work exploring the case of asymmetric pass-through, or DCP (see Gopinath [2016] and Casas et al. [2016]). An important question is which direction monetary policy will take in the country which issues the dominant currency, when facing a capital inflow with currency overvaluation or undervaluation.
References


[27] Devereux, Michael B. and Changhua Yu [2016], “Monetary and Exchange Rate Policy with Endogenous Financial Constraints,” mimeo, University of British Columbia.


The parameter values are as follows: $\eta=0, \phi=\sigma=1, aH=.75, \beta=.99, \alpha=.75, b=3.$