Currency crises
A reconsideration of the classical model

Giancarlo Corsetti
University of Cambridge

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Outline

1 Model
- Residents’ problem
- Government budget
- Fiscal requirement for a permanent currency peg
- Shocks causing fiscal imbalances
- Fiscal policy
- Monetary policy

2 The dynamic of the crisis
- The equilibrium jump in the exchange rate in response to shocks
- The value of long-term debt as shock absorber
- The “natural collapse” of a peg
- The “shadow interest rate” and the timing of a collapse
- A role for international reserves?
Krugman’s model of currency crises is one of the most famous pieces in international economics.

Krugman draws on work by Henderson and Salant (HS), modelling speculative attacks on gold when the its price is pegged to some official price by an authority endowed with a finite stock of gold reserves.

HS makes the point that a seemingly random run on gold—exhausting the stock of reserve—is actually a (unique) equilibrium outcome. The model assumes that, absent the peg, the equilibrium price of gold, defined as the shadow price of gold, would be trended upward. They show that a run occurs as soon as this shadow price is above the official price.

Similarly, Krugman assumes that, because of deteriorating fiscal conditions, the ‘equilibrium’ shadow exchange rate suffers a trend depreciation. He shows that a speculative attack on the currency peg occurs as soon as the shadow rate is weaker than the parity at which the exchange rate is pegged.

This model is a classic. However, its structure does not connect the fiscal and the monetary side of the story. This is done in what follows, drawing on Corsetti and Mackowiak, European Economic Review 2005.
Consider a small open economy, populated by many identical individuals, endowed with an exogenous stream of output.

Residents can borrow in three assets: one period bonds denominated in either home currency and in foreign currency, $B$ and $B^*$, as well as in a perpetuity (very long term bond) $L$, paying 1 unit of domestic currency in each period. Note: all these are denominated in nominal terms and are free of (nominal) default.

In addition the government also borrows in a foreign-currency (dollar) denominated bond $F^*$.

There is no uncertainty. So the portfolio problem is indeterminate. However, in equilibrium, prices of different bonds need to obey no arbitrage/equilibrium conditions, hence the Uncovered Interest Parity condition UIP holds.

Prices are flexible.

There is a single good in the world economy: purchasing power parity holds at each point in time, $P_t = E_t P_t^*$.
Residents’ problem

Let $Y$ and $C$ denote endowment and consumption of a output, respectively, and $\tau$ denote real lump-sum taxes (or transfers, if negative).
The representative individual in the economy maximizes:

$$\sum_{t=0}^{\infty} \beta^t \ln C_t$$

subject to:

$$\frac{Q_t L_t}{P_t} + \frac{B_t}{P_t} + \frac{E_t B^*_t}{P_t} \leq \frac{(Q_t + 1) L_{t-1}}{P_t} + \frac{(1 + i_{t-1}) B_{t-1}}{P_t} + \frac{E_t (1 + r) B^*_{t-1}}{P_t} + Y_t - \tau_t - C_t$$

in every period, as well as to a no Ponzi game condition:

$$\lim_{t \to \infty} \left( \frac{1}{1 + r} \right)^t \left( \frac{Q_t L_t}{P_t} + \frac{B_t}{P_t} + \frac{E_t B^*_t}{P_t} \right) \geq 0$$

Consumers cannot accumulate debt indefinitely at a rate faster than the interest rate on their debt (cannot repay interest with further borrowing forever).
Optimal consumption and portfolio plan

The first order conditions (foc) with respect to $B^*$ and $B$ imply UIP:

$$1 + i_t = (1 + r) \frac{E_{t+1}}{E_t}$$  \hspace{1cm} (2)

and the foc with respect to $L$ implies:

$$(1 + i_t) = \frac{1 + Q_{t+1}}{Q_t}$$  \hspace{1cm} (3)

Solving (3) forward, the price of the perpetuity is the present discounted value of the infinite stream of nominal coupon equal to 1, discounted using current and future nominal interest rates:

$$Q_t = \sum_{s=0}^{\infty} \prod_{k=0}^{s} \left( \frac{1}{1 + i_{t+k}} \right)$$  \hspace{1cm} (4)

The “transversality condition” of the agent’s problem states that it would be irrational for non-satiated consumer to accumulate asset faster than the interest rate, so together with the no-Ponzi game condition we have:

$$\lim_{t \to \infty} \left( \frac{1}{1 + r} \right)^t \left( \frac{Q_t L_t}{P_t} + \frac{B_t}{P_t} + \frac{E_t B_t^*}{P_t} \right) = 0$$  \hspace{1cm} (5)
Government budget

The government can issue single-period dollar bonds, denominated by $F^*$. Including these, the period-by-period consolidated government budget identity is:

$$\frac{Q_t L_t}{P_t} + \frac{B_t}{P_t} + \frac{\mathcal{E}_t F^*_t}{P_t} = \frac{(Q_t + 1) L_{t-1}}{P_t} + \frac{(1 + i_{t-1}) B_{t-1}}{P_t} + \frac{\mathcal{E}_t (1 + r) F^*_{t-1}}{P_t} - \tau_t$$  (6)

Note that the government borrows in dollars at the world interest rate $r$, implying that it does not default on its dollar bonds. Using the individual’s first order conditions and the following terminal condition:

$$\lim_{t \to \infty} \left( \frac{1}{1 + r} \right)^t \left( \frac{Q_t L_t}{P_t} + \frac{B_t}{P_t} + \frac{\mathcal{E}_t F^*_t}{P_t} \right) = 0$$  (7)

we obtain a solved-forward version of the government budget constraint (6):

$$\frac{(Q_t + 1) L_{t-1} + (1 + i_{t-1}) B_{t-1}}{P_t} + \frac{\mathcal{E}_t (1 + r) F^*_{t-1}}{P_t} = \sum_{s=0}^{\infty} \left( \frac{1}{1 + r} \right)^s \tau_{t+s}$$  (8)

In equilibrium, the real value of public liabilities (LHS) equals the present discounted value of future real primary surpluses (RHS).
A competitive equilibrium in this small open-economy is a specification for a time path of the vector \( \{ Y, r, C, B^*, B, L, F^*, \tau, i, Q, \mathcal{E}, P^*, P \} \) such that:

1. when the representative individual takes the time path of \( \{ Y, r, \tau, i, Q, \mathcal{E}, P^*, P \} \) as given, \( \{ C, B^*, B, L \} \) solves her optimum problem;
2. the government budget constraint (8) holds.

Recall that the government chooses \( F^* \), private agents choose \( B^* \).

We now show that, if the monetary authorities fix the exchange rate, for the peg to be sustainable as an equilibrium, there are important restrictions on fiscal policy.
Fiscal requirement for a permanent currency peg

Suppose that the foreign price level is equal to a constant, $P^*$, and that the government fixes the exchange rate at $E$. For a fixed exchange rate to be sustainable, fiscal policy must be consistent with the government’s solved-forward budget constraint (8) holding at $E$. Let $\{\tau_{t+s}\}_{s=0}^{\infty}$ be a path of primary surpluses such that (8) holds given $P^*$ and initial debt $(B_{t-1}, L_{t-1}$ and $F_{t-1}^*)$ with the exchange rate permanently fixed at $E$, that is $E_{t+s} = E$, $s \geq 0$. By definition, then, $\{\tau_{t+s}\}_{s=0}^{\infty}$ is a sequence such that:

$$\sum_{s=0}^{\infty} \left( \frac{1}{1+r} \right)^s \tau_{t+s} = \frac{(1 + 1/r) L_{t-1} + (1 + r) B_{t-1}}{E P^*_\alpha} + \frac{(1 + r) F_{t-1}^*}{P^*_\alpha}$$

To satisfy the above condition, a sequence of $\tau$’s must eventually imply a strong feedback from debt to $\tau$’s: if debt accumulates in response to shocks hitting the economy, primary surpluses must rise sufficiently—enough to back debt in full, given $E$. The literature refers to this fiscal policy as **passive or Ricardian**: the government adjust its policy taking the equilibrium path of prices $P_t = E_t P^*_t$ as given.
Shocks causing fiscal imbalances: nominal and real

Without loss of generality, suppose the foreign price level changes in period $t$ to a new, permanent level $P^*_\beta$ where $P^*_\beta < P^*_\alpha$—an unexpected foreign deflation. Given a fixed exchange rate parity, the domestic price level falls correspondingly. Hence, on impact, the shock increases the real value of both nominal and dollar debt.

In words: Foreign deflation increases the quantity of the real commodity the government promises to bondholders, regardless of whether their claims are denominated in domestic or in foreign currency.

Given $\{\bar{\tau}_{t+s}\}_{s=0}^{\infty}$, the solved-forward government budget constraint (8) in period $t$ is violated:

$$\frac{(Q_t + 1) L_{t-1} + (1 + r) B_{t-1}}{\bar{E} P^*_\beta} + \frac{(1 + r) F^*_{t-1}}{P^*_\beta} > \left[ \sum_{s=0}^{\infty} \left( \frac{1}{1 + r} \right)^s \bar{\tau}_{t+s} \right]$$
The government presumably undertakes some measures in reaction to the shock. Let $\Delta$ denote the resulting deficit: $\Delta < 0$ corresponds to improvement (lower deficit) after reform, while $\Delta > 0$ to deterioration beyond the immediate effect of the shock.

Let’s assume that from time $T_d$ onwards primary surpluses fall by a constant $d$ (d stands for deficit, a negative $d$ is an improvement):

$$\Delta \equiv \frac{(1 + r) d}{r} \left( \frac{1}{1 + r} \right)^{T_d - t}$$

Note that the balance may start deteriorating in the future—setting $T_d \geq t$ allows for this possibility.
Policy rules and shocks causing fiscal imbalances

Since the fiscal adjustment is insufficient to offset the shock (it may actually exacerbate the fiscal problem), at the new price level we have:

\[
\left[\frac{(1 + 1/r) L_{t-1} + (1 + r) B_{t-1}}{\bar{E} P^*_\beta} + \frac{(1 + r) F^*_{t-1}}{P^*_\beta}\right] > \left[\sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^s \bar{T}_{t+s}\right] - \Delta
\]

Note that above can hold as an equality if \( E_t \) adjusts:

\[
\frac{(Q_t + 1) L_{t-1} + (1 + r) B_{t-1}}{\bar{E}_t P^*_\beta} + \frac{(1 + r) F^*_{t-1}}{P^*_\beta} = \left[\sum_{s=0}^{\infty} \left(\frac{1}{1 + r}\right)^s \bar{T}_{t+s}\right] - \Delta \quad (10)
\]

This is because a depreciation would create instantaneous imported inflation, which would reduce the real value of outstanding liabilities (for given real primary surpluses/deficits).
Monetary policy

To complete the model, we need to specify monetary policy. Obviously, so long as the exchange rate is fixed at $\bar{E}$, the nominal interest rate is determined by the uncovered interest rate parity (2) evaluated at $E_t = \bar{E}$.

But what policy rule will be followed after the country devalues? Posit that, in the event of a devaluation at time $\tilde{T}$, the country abandons the peg forever and in each subsequent period $\tilde{T} + s$ the monetary authorities adopt an interest rate rule:

$$1 + i_{\tilde{T}+s} = \phi_0 + \phi_1 \frac{P_{\tilde{T}+s}}{P_{\tilde{T}+s-1}} = \phi_0 + \phi_1 \frac{E_{\tilde{T}+s}}{E_{\tilde{T}+s-1}}$$

(11)

According to this rule, the (gross) nominal interest rate is a linear function of the inflation rate that, given $P^*_\beta$, is equal to the (gross) rate of currency depreciation. When $\beta \phi_1 < 1$, policy makes $i$ react weakly to depreciation (or inflation). Note that the reaction function (11) is consistent e.g. with inflation targeting in the post-devaluation period.

For tractability, posit $\phi_1 = 0$, so that in the post-devaluation period the interest rate is constant and equal to $1 + i^p_{\tilde{T}+s} = \phi_0$, where $p$ stands for post-devaluation. What follows applies also in the general case, but the algebra is messy.
The shadow or equilibrium exchange rate in response to a shock causing a fiscal imbalance

Knowing the size of fiscal imbalance and the post-devaluation monetary policy, we can solve for the equilibrium devaluation rate \( \left( \mathcal{E}_t / \mathcal{E} \right) \).
To do so, we make use of the solved-forward government budget constraint — combining (9) with (10).

- Let \( \pi_t^* \) denote unexpected foreign inflation at time \( t \): \( \pi_t^* \equiv \frac{P^* - P}{P^*} \); note that at time \( t \), \( \pi_t^* \leq 0 \). It is zero afterwards.
- Let \( B_{t-1} \) denote the initial stock of short-term debt in domestic currency (and \( F^*_{t-1} \) — the stock of debt in foreign currency), expressed as ratios of long-term debt:

\[
B_{t-1} \equiv \frac{(1 + r) B_{t-1}}{L_{t-1}} \\
F^*_{t-1} \equiv \frac{(1 + r) \mathcal{E} F^*_{t-1}}{L_{t-1}}
\]
Finally, define $\ell_{t-1}$ as the real value of long-term debt if no devaluation occurs, i.e. $\ell_{t-1} \equiv \frac{L_{t-1}}{E(1 + \pi_t^*) P^*_\alpha}$.

Using this new notation, and recalling that $i^p$ denotes the post-devaluation constant interest rate, we can write the unique solution for the equilibrium devaluation rate as follows:

$$
\frac{E_t}{E} = \frac{1 + i^p}{i_p} + B_{t-1} + \frac{(1 + \pi_t^*) (1 + r)}{r} + (1 + \pi_t^*) B_{t-1} + \pi_t^* F^*_{t-1} - \frac{\Delta}{\ell_{t-1}}
$$

The jump in the shadow exchange rate is increasing in the size of the fiscal imbalance: $\left(\frac{E_t}{E}\right)$ is decreasing in $\pi_t^* (P^*_\beta)$ and increasing in $\Delta$. The exchange rate change is determined both by the size of the shock and by the policy reaction to it.
The value of long-term debt as shock absorber

We have seen that a fiscal shock causes a jump in the equilibrium devaluation rate. However, this does not mean that the currency immediately devalues. To see how this is possible, combine (9) and (10) with \( E_t = \bar{E} \) and solve for the (unique) equilibrium perpetuity price in period \( t \):

\[
Q_t = (1 + \pi_t^*) \left( \frac{1}{r} - \Delta \frac{\bar{E}P^*_t}{L_{t-1}} \right) + \pi_t^*(B_{t-1} + F^*_t - 1) \geq 0. \tag{13}
\]

If the exchange rate does not adjust immediately, provided that \( L_{t-1} \) is large enough relative to \( \Delta \) (so that \( Q \geq 0 \)), equilibrium can be guaranteed by a downward jump in the long-term bond price. This jump ensures that the solved-forward government budget constraint holds with \( E_t = \bar{E} \).

In words: if the fiscal shock does not lead to immediate devaluation, it nonetheless causes a fall in the price of the government long-term debt. The stock of long-term debt in domestic currency, \( L_{t-1} \), acts as a cushion to the government: the larger this stock, the smaller jump in the bond price.

But while the adjustment in \( Q_t \) may make the peg temporarily viable, the fiscal imbalance must eventually lead to depreciation.
The “natural collapse” of a peg

Let \( \tilde{T} > t \) denote the date at which the government abandons the peg. What follows shows that \( T \) must be finite.

Using (2) and (3) together with the post-devaluation constant policy rate \( i_{\tilde{T}+s} = i^p \), we can rewrite the perpetuity price in any period \( t \) before \( \tilde{T} \) as follows:

\[
Q_t = \frac{1}{r} + \left( \frac{1}{1 + r} \right)^{\tilde{T} - t - 1} \left( \frac{1 + (1/i^p)}{(1 + r)(E_{\tilde{T}}/E)} - \frac{1}{r} \right)
\]  \hspace{1cm} (14)

Putting together (13) and (14) yields the unique solution for the equilibrium devaluation rate \( (E_{\tilde{T}}/E) \):

\[
\frac{E_{\tilde{T}}}{E} = \frac{1 + i^p}{i^p} \frac{1 + r}{r} - (1 + r)^{\tilde{T} - t} \left[ \frac{\Delta}{\ell_{t-1}} - \pi^*_t (1 + B_{t-1} + F^*_t) \right]
\]  \hspace{1cm} (15)
The “natural collapse” of a peg

Recall that in calculating \( \frac{\mathcal{E}_T}{\mathcal{E}} \), we assumed that \( \Delta \) is not too large. In particular, looking at the denominator of the previous expression:

\[
\frac{1 + r}{r} > (1 + r)^{\bar{T} - t} \left[ \frac{\Delta}{\ell_{t-1}} - \pi^*_t (1 + B_{t-1} + \mathcal{F}^*_t) \right]
\]

(16)

In words: \( \Delta \) is no larger than the present value of the maximum “wealth transfer” from long-term bond holders. From the above formula, it is apparent that, for the inequality to hold, \( \bar{T} \) cannot be too large. Hence equilibrium implies that a currency collapse must eventually occur in finite time. The latest date beyond which the collapse cannot be delayed (referred to as the “natural collapse” of the peg) is given by the critical \( \hat{T} \) at which the sign of the above inequality switches:

\[
\hat{T} \simeq t + \frac{r - \log r}{r} - \frac{1}{r} \log \left[ \frac{\Delta}{\ell_{t-1}} - \pi^*_t (1 + B_{t-1} + \mathcal{F}^*_t) \right]
\]

(17)

The larger the stock of long-term nominal debt \( L_{t-1} \), the larger the shock absorbing capacity of the government budget, the longer can the government keep the exchange rate fixed.
The “shadow interest rate” and the timing of a collapse

The “natural collapse” depends on the interest rate policy that the monetary authorities follow after the devaluation. For instance, focusing on the case in which the constant, post-devaluation policy rate is close to zero $\pi^* \rightarrow 0$, the above expression simplifies to

$$\hat{T} \simeq t + \frac{r - \log r}{r} - \frac{1}{r} \log \Delta + \frac{1}{r \log \left( \frac{L_{t-1}}{\bar{E}P^*_{\alpha}} \right)}$$

The question is now: when will a crisis actually occur?

To answer this question, observe that in the period preceding a collapse (hence before monetary authorities set $i = i^p$), investors will price the coming devaluation: while the peg is still holding, expectations of depreciation will raise the one-period equilibrium domestic nominal interest rate above the foreign one (by UIP).

We have seen that the longer the government waits before devaluing, the higher the devaluation rate: hence the higher the jump in interest rates one period before the peg collapses. In Figure 2 below, this is illustrated by the “shadow nominal rate”, i.e., the equilibrium nominal rate that would prevail at each point in time, if investors expect a crisis one period ahead.
Interest rate policy and the timing of a collapse

Again: the key observation is that, as long as it operates under a peg, the central bank needs to accommodate whatever interest rate is consistent with the fixed exchange rate parity (in our economy, this is implied by the UIP condition).

Posit that, for some reason external to the model, the government finds high interest rates costly to accept. Specifically, assume that, realistically, there is an upper bound on the interest rate that policymakers are unwilling to cross: the government will defend the peg as long as doing so will not lead the country to operate at interest rates above this critical level.

This means that the government abandons the peg in the first period $T^{\text{Crisis}} < \tilde{T}$ such that keeping the exchange rate fixed would cause the interest rate to rise to or above $\bar{i}$ at $T^{\text{Crisis}}$. Then, this means that period $(T^{\text{Crisis}} - 1)$ is the last period in which the interest rate is below the policy-specified threshold:

$$T^{\text{Crisis}} = \min s \quad \text{such that} \quad i_s \geq \bar{i} \quad (18)$$
Figure 1: The dynamics of the exchange rate - an illustrative example

- Realized exchange rate in equilibrium
- Shadow exchange rate

News arrives (period t)
Devaluation takes place

The interest rate goes up one period before devaluation (the last period in which the shadow interest rate is below the bound set by policy)

Figure 2: The dynamics of the short-term nominal interest rate - an illustrative example

- Realized nominal interest rate in equilibrium
- Shadow nominal interest rate
- Upper bound set by policy

The interest rate goes up one period before devaluation (the last period in which the shadow interest rate is below the bound set by policy)
Interest rates, money demand and runs on international reserves

As shown in the figure to follow, the upper bound on the interest rate uniquely determines the timing of the crisis and the devaluation rate. The higher this bound, the larger the collapse.

Let’s trace the implications for “international reserves”.

- We have seen that in a last “attempt” to defend the peg, the central bank accommodates the high equilibrium rate.
- Add to the model a conventional money demand function $M(y, i)$—for simplicity, do so without changing consumers’ problem.
- Since $M(y, i)$ is a negative function of the nominal interest rate, the jump in $i$ leads to a contraction in money demand. This requires the central bank to sell its holding of either domestic debt or international reserves, the latter at the pegged exchange rate.
- Krugman assumes that the contraction in money demand drives down reserves.
Interest rates, money demand and runs on international reserves

A key lesson from the Krugman model is that, when a currency peg is bound to collapse for “natural causes”, the timing of a speculative attack is determined by the size of a finite stock of international reserves. According to the Krugman model, this stock is depleted via a contraction in the demand for domestic money, responding to an interest rate hike in anticipation of a devaluation. The largest the stock of reserves, the deepest the contraction in demand for money required to exhaust it, the highest the equilibrium interest rate in the period preceding the collapse.

The analysis above is making the same point, but instead of focusing on a finite stock of international reserves, call attention on the constraint on the maximum acceptable interest rate. The two map into each other one-to-one: the higher the stock of reserves in Krugman, the higher the upper bound on the interest rate in our model.
Conclusions

Why revisiting Krugman? The motivation is as follows:

- the new version explicitly models the fiscal imbalance at the root of the currency collapse;
- the new model is closer to recent macroeconomic literature and policy analysis, which frame monetary policy in terms of interest rate rules;
- the new framework resolves two paradoxes of the Krugman model, as classic as the model itself:
  1. in Krugman (1979), despite the shock, the exchange rate is predicted to appreciate if the peg is abandoned one period before the collapse $T^{\text{Crisis}}$: very strange, given that currency instability is caused by a fiscal imbalance.
  2. In Krugman (1979), the equilibrium condition requires investors to buy reserves at the already devalued exchange rate. In reality, investors run on reserves taking advantage of the fact that the central bank sell them at the pegged rate before the collapse. In the model above, the contraction of money demand (which may be accommodated by selling reserves) occurs when the currency is still pegged.