The International Dimension of Productivity and Demand Shocks in the US Economy

Giancarlo Corsetti
Cambridge University, University of Rome III and CEPR
Luca Dedola
European Central Bank and CEPR
Sylvain Leduc
Federal Reserve Bank of San Francisco

January 2013

Abstract

This paper analyzes the cross-country effects of productivity and demand disturbances in the US identified with sign restrictions based on standard theory. Productivity gains in US manufacturing increase US consumption and investment vis-à-vis foreign countries, resulting in a trade deficit and higher international prices of US goods, despite the rise in their supply. Financial adjustment works via a higher global value of US equities, real dollar appreciation, and an expansion of US gross foreign liabilities as well as assets. Positive demand shocks to US manufacturing also increase investment and cause a real dollar appreciation, but have limited effects on the trade balance and net foreign assets. Our findings emphasize the importance for macroeconomic interdependence of endogenous fluctuations in aggregate demand across countries in response to business cycle shocks. Our empirical characterization of macroeconomic interdependence emphasizes the key role of endogenous fluctuations in aggregate demand across countries in response to business cycle shocks.

JEL classification: F32, F41, F42

Keywords: International transmission mechanism, structural VARs, sign restrictions, US current account, dollar real exchange rate.

---

1 We thank the editor Fabio Canova and three anonymous referees, Larry Christiano, Martin Eichenbaum, Emanuel Farhi, Fabio Ghironi, Wouter den Haan, Andrea Raffo, Sergio Rebelo, Silvana Tenreyro, Martin Uribe as well as seminar participants at a number of institutions and conferences. We are grateful to Flavia Corneli, Kerstin Holzheu and Lenno Uusküla for outstanding research assistance. Corsetti’s work on this paper was part of the Pierre Werner Chair Programme on Monetary Union, at the Robert Schuman Centre of the European University Institute. The views expressed here are those of the authors and do not necessarily reflect the positions of the ECB, the Board of Governors of the Federal Reserve System, or any other institutions with which the authors are affiliated.
1 Introduction

What are the consequences of US business cycle shocks for the US trade balance and capital flows, the international value of the dollar and the dynamics of US foreign assets and liabilities? In this paper we carry out a study of the international dimension of US business cycle impulses by identifying structural shocks to productivity and demand in US manufacturing, and tracing their effects on a broad range of variables, encompassing both real and financial channels of the international transmission.

While structural analysis is essential to understanding international macroeconomic interdependence, in the large literature on the macroeconomic effects of US shocks only a few studies have gone beyond a closed economy context, extending the analysis to their international transmission. Moreover, these studies restrict their focus on the effects of policy innovations. In this paper, we bring structural investigation to bear on the transmission of demand and supply shocks in the US, providing comprehensive conditional evidence on their open-economy effects. Our findings shed light on the empirical content of a broad range of open-economy models, with a set of novel results (especially on relative prices and financial variables) that provide empirical guidance for theoretical modelling.

We identify productivity and demand shocks in structural VARs in a novel way, adopting the sign-restriction methodology proposed by Canova and De Nicoló (2002) and Uhlig (2005). We impose theory-based restrictions on the sign of the impulse responses of a small subset of variables, consistent with the intuitive idea that prices and quantities of US manufacturing relative to US non-manufacturing output should move in the same direction in the case of demand shocks, and in opposite direction in the case of productivity shocks. Our focus is deliberately on sectoral shocks — whereas manufacturing accounts for a large share of US international trade. As discussed in detail in the text, focusing on shocks to one sector facilitates identification, and avoid potentially difficult issues in the interpretation of the results. Using a rich version of the workhorse open-economy monetary model with traded and nontraded goods, we show that our identifying restrictions hold across a wide array of

2See e.g. the literature on the effects of technology shocks on the US economy, surveyed by Galí and Rabanal (2005).

3See e.g. Eichenbaum and Evans (1995) for monetary policy shocks, and Roubini and Kim (2008) for fiscal policy shocks. As an exception, in notable early work, Clarida and Galí (1994) used long-run restrictions to identify aggregate demand and supply shocks and their effects on the real exchange rate, and cross-country GDP and inflation differentials for the US vis-à-vis the other G7 countries.

4The method of identifying monetary policy shocks using sign restrictions on impulse responses has been introduced by Faust (1998) and Uhlig (2005); Faust and Rogers (2003) and Scholl and Uhlig (2007) provide open-economy applications. See also, among others, Canova and De Nicoló (2002), Mountford (2005), Canova and Pappa (2006), Dedola and Neri (2007), Mountford and Uhlig (2009), and Enders et al. (2011), and Canova and Paustian (2011).

5In particular, our study of the US economy, identify the effects of the two identified shocks, on as many as 17 variables, systematically documenting real and financial channels of transmission.
theoretical specifications studied in the literature.

In earlier work (Corsetti, Dedola and Leduc 2008c) we have used long-run restrictions as in Galí (1999) to identify technology shocks in the manufacturing sector in a sample of five G7 countries, and carry out a comparative study of their effects focusing on seven macro variables. The novel and straightforward identification strategy developed here helps us address many of the shortcomings arising from the adoption of long-run restrictions to identifying sectoral shocks. In addition, it allows us to analyze demand as well as productivity shocks. While showing that, for the US, our previous results are robust to our new identification strategy—a result we find reassuring—we document the effects of our two identified shocks on as many as seventeen variables. Moreover, we analyze not only the real, but also the financial dimension of the international transmission mechanism.

Our main results are as follows. As regards productivity shocks to US manufacturing, we find that they have significant macroeconomic effects. Positive shocks persistently increase US aggregate consumption and investment relative to the rest of the world, raising real imports and worsening the US trade balance. Our empirical model thus provides conditional evidence consistent with key predictions of the intertemporal-trade approach to the current account, stressing consumption smoothing and fluctuations in absorption. While raising the relative supply of US manufacturing output, however, positive productivity shocks result in higher international prices of US goods (proxied by both an export-deflator-based and a PPI-based real exchange rate). Correspondingly, the real dollar (CPI-based) exchange rate appreciates.

By the same token, we document substantial international repercussions of US productivity shocks through financial channels. In addition to causing real appreciation, these shocks sharply raise the value of the US stock market relative to an aggregate index of foreign markets, and open a positive nominal interest differential in favor of the US over time. Using the series of valuation-adjusted US foreign assets and liabilities computed by Gourinchas and Rey (2007), we find that asymmetric productivity gains worsen the US net foreign asset position relative to GDP, on impact by more than the deterioration in net trade. In turn, the fall in US net foreign asset position corresponds to an increase in both US gross external assets and liabilities.

As regards demand shocks to US manufacturing, we find that they generally have the expected qualitative effects, but, quantitatively, they tend to be far less consequential than productivity shocks, for several aggregate variables. In our identification, a positive shock to demand for US manufacturing is assumed to lead to an increase in both the relative price and output of US manufacturing. In response to such shock, US absorption rises, reflecting an increase in aggregate investment, while international relative prices strengthen significantly. Despite the rise in investment and the real appreciation, however, we find
limited crowding out of net exports. The response of both real and financial variables —
including consumption, trade flows, gross and net external asset positions, stock prices and
interest differentials — is rather subdued.

Finally, whether conditional on (positive) productivity or demand shocks in the man-
ufacturing sector, we find real exchange rate appreciation that is to a large extent driven
by an increase in the international prices of US manufactures — a result that confirms and
complements the unconditional evidence in Engel (1999).

These findings provide support for the international transmission mechanism in stan-
dard open-economy models, but also questions specific aspects of it. First, contrary to the
conventional wisdom rooted in the Mundell-Fleming tradition, real demand shocks for US
manufacturing which raise domestic absorption and appreciate the dollar in real terms, do
not produce appreciable crowding-out effects on net trade.\(^6\) Second, in contrast with the
predictions by leading modern open-economy models (see e.g. see Backus, Kehoe and Kyd-
land 1994 and Stockman and Tesar 1995), domestic productivity gains in the US causing
an expansion in a large chunk of US tradable output relative to the rest of the world are
accompanied by an appreciation of US international relative prices. As real appreciation
prevents other countries from benefitting from the US-based productivity improvement in
terms of cheaper imports, the efficient relative-price channel envisioned by most standard
business cycle models\(^7\) does not appear to be operative in the data.

By the same token, our finding that US equity prices markedly increase relative to equity
prices abroad, suggests that widely traded assets provide opportunities for diversifying risk
arising from productivity fluctuations. Yet these opportunities appear to be largely unex-
pected: in response to productivity shocks, US relative consumption rises in tandem with its
relative price (the real exchange rate), providing conditional evidence at odds with standard
conditions for efficient diversification. To explore whether these results could be attributed
to insufficient diversification, we calibrate portfolio shares in our theoretical model imposing
the observed degree of home bias in aggregate US equity portfolio, rather than using the
optimal portfolio composition implied by the model itself. We show that, once we do so, our
theoretical model’s predictions are indeed in line with the above empirical findings.

Overall, our study emphasizes the need to deepen our understanding of the endogenous
response of cross-country aggregate demand to business cycle impulses. The traditional
Mundell-Fleming-Dornbusch (MFD) framework stresses asymmetric real demand distur-
bances treating them as exogenous. As amply discussed in the literature (see e.g. Dorn-

\(^6\) According to the textbook Mundell-Fleming model, under flexible exchange rates, a shock to real demand
(moving the IS curve) unambiguously appreciates the currency in real terms. It also results in a deterioration
of net exports, provided that the origin of the shock is of prevailing domestic origin (e.g. it raises the so-called
autonomous components of investment and/or consumption spending).

\(^7\) See e.g. Cole and Obstfeld (1991).
busch 1980), however, many sharp predictions in the MFD tradition obtain only because this framework does not explicitly account for any endogenous changes in wealth, which in some instance may largely offset the effects of exogenous demand disturbances postulated in the model.\textsuperscript{8}

Differently from the traditional MFD analysis, modern open-economy theory makes it clear that structural impulses, including supply shocks, can generate endogenous movements in wealth and thus demand across countries.\textsuperscript{9} In line with our findings, for instance, recent theoretical literature has shown that, in incomplete market economies, supply disturbances lead to wealth and demand responses disproportionately concentrated in the economy experiencing a shock. Namely, in Ghironi and Melitz (2005), asymmetric demand effects from a positive technology shock are associated with an appreciation of the terms of trade and inefficient adjustment at the extensive margin.\textsuperscript{10} In Corsetti et al. (2008a), persistent productivity and output booms in a large, relatively closed economy like the United States, can induce unbalanced movements in cross-country wealth and demand that cause both the trade balance to deteriorate and the international price of domestic goods to appreciate.

The paper is organized as follows. Section 2 describes our identification strategy and discusses its theoretical foundations. Section 3 describes the data and the empirical methodology. Section 4 reports and analyzes in detail our main findings, while Section 5 presents some sensitivity analysis. Section 6 concludes.

\section{Identifying productivity and demand shocks via sign restrictions}

In line with the literature using sign restrictions, our empirical strategy consists of selecting a minimal set of common theoretical predictions by a large class of models, restricting the direction of the responses of key variables to a particular shock of interest. In our identification scheme, on the one hand, we are careful to restrict only variables for which theory’s predictions are unambiguous and fairly uncontroversial. On the other hand, we are also

\textsuperscript{8}This issue is at the heart of the debate on the Harberger-Laursen-Metzler effect, revolving around the relationship between aggregate spending, the current account and international relative prices — see e.g. Obstfeld (1982) and Svensson and Razin (1983).

\textsuperscript{9}Endogenous wealth divergences are ruled out by assumption in the open-economy literature positing complete markets or a high degree of cross-border risk sharing through efficient relative price movements, including many seminal contributions to the modern international real business cycle literature — e.g. see Backus, Kehoe and Kydland (1994) and Stockman and Tesar (1995) — as well as to the modern with literature with nominal rigidities — e.g., see Obstfeld and Rogoff (2000) and Chari, Kehoe and McGrattan (2002).

\textsuperscript{10}Ghironi and Melitz (2005) refers to the ‘terms of labor’, that is, relative cost of effective units of labor across countries, which is proportional to the terms of trade.
careful not to restrict variables for which theoretical predictions differ across models, as to allow our exercise to provide insight on theoretical debates.

To verify that our restrictions are robustly grounded in economic theory, we specify a standard open economy model that encompasses many of the features emphasized in the literature – traded and nontraded goods, home bias in domestic demand, price rigidities, assuming either local currency pricing (LCP) or producer currency pricing (PCP), for exports, and alternative policy rules. Drawing on Canova and Paustian (2008), we show that our identifying restrictions are met with a high degree of confidence by drawing model responses to a supply and a demand shock for a large number of parameter configurations.

2.1 Identification strategy

Our strategy builds on a straightforward idea: supply-side shifts should move relative prices and quantities in opposite directions, while demand shifts should move them in the same direction. Since this idea is especially powerful when applied to sector- or industry-specific shocks, a natural focus for studies of the international transmission mechanism is on the manufacturing sector, which we use as a proxy for the tradable sector. In standard (open-economy) multisector models, both productivity and demand shocks in one sector increase its output relative to overall GDP; however, productivity gains unambiguously decrease the sectoral price in terms of the aggregate of the other goods produced in the economy; demand shocks unambiguously increase it. Thus, we can use the domestic relative price of manufacturing in terms of the other domestically produced goods as the key price variable in our identification scheme. Note that, according to our choice criteria, the international relative price of domestic manufacturing should not be selected as the price variable. This is because, as discussed in the introduction, theory’s predictions on the response of international prices to shocks depend on the amount of cross-country insurance and more generally on the international transmission mechanism — whose investigation is the main objective of our study.

Focusing on manufacturing not only facilitates identification but also avoids potentially

---

\(^{11}\)It is more problematic to identify demand and supply shifts by looking at movements in aggregate quantities and nominal prices in *levels*, rather than in relative terms. To wit, identification using price levels requires conditioning on specific monetary policy responses to shocks.

\(^{12}\)This principle underlies both the Baumol effect and the Harrod-Balassa-Samuelson effect. The textbook rendition of the HBS effect typically assumes country-specific nontrads and a homogeneous tradable good (so that terms of trade are constant), and abstracts from general equilibrium effects, in particular those stemming from asset trading. Under these simplifying assumptions, an increase in the domestic relative price of nontraders unambiguously translates into real currency appreciation. As discussed in the next subsection, however, when national tradables are differentiated, and the role of cross-country risk insurance is taken into account, the theoretical prediction that productivity gains in tradables lowers their price in terms of domestic nontrads does not necessarily translate into real exchange rate appreciation.
difficult issues in the interpretation of the results, relative to the alternative of analyzing economy-wide shocks with unspecified industry origin. In this alternative, the response of many macro (internal and external) variables, including relative prices, is bound to be sensitive to the distribution of shocks across sectors. Interpreting empirical results would then require additional and possibly controversial assumptions about the relative importance of aggregate disturbances in each sector.\textsuperscript{13}

By the same token, consistent with our interest on the international transmission, our analysis naturally focuses on those shocks hitting the United States asymmetrically vis-à-vis the rest of the world. Failure to focus on asymmetric disturbances would raise issues in interpreting shocks of unspecified geographic origin, similar to the one discussed above.\textsuperscript{14} Our identification scheme is detailed below.

**Productivity in US manufacturing** The first shock under consideration consists of supply shocks increasing labor productivity in the US manufacturing sector, relative to that in the rest of the world. To identify these shocks we postulate a set of four restrictions. Positive supply shocks should: (1) raise (the log of) manufacturing output relative to aggregate output in the US; (2) lower the domestic price of US manufacturing relative to US non-manufacturing production; (3) raise US manufacturing output relative to Foreign manufacturing output; and (4) raise US labor productivity relative to Foreign labor productivity in manufacturing.

As explained earlier, the theoretical underpinning of restrictions (1) and (2) is that supply shifts move relative prices and quantities in opposite directions. Restriction (3) isolates shocks with stronger US-specific effects. The last restriction ensures that our identified supply shocks are associated with an increase in relative labor productivity, as predicted by the international real business cycle (IRBC) literature based on standard technology shocks. All other variables included in our analysis, namely, aggregate consumption and investment, trade variables, capital flows, international relative prices and asset prices, are left unconstrained, as to trace the external consequences of productivity shocks in the data.

\textsuperscript{13}To wit: suppose that, in response to economy-wide shocks, the empirical model would suggest a positive association between the level of the US economy-wide labor productivity and a terms-of-trade deterioration. It would be quite difficult to infer that the depreciation is evidence in favor of a particular transmission mechanism without knowing whether the productivity increase is concentrated in tradables or nontradables.

\textsuperscript{14}Without controlling for global and asymmetric disturbances, the interpretation of the international repercussions of identified shocks would again require auxiliary assumptions about their distribution and consequences across countries. For instance, suppose the empirical model suggests a positive association between the level of US labor productivity and the US trade deficit. Without controlling for movements in foreign productivity, it would not be possible to infer that this is evidence in support of the intertemporal approach to the current account — a point stressed by Glick and Rogoff (1995).
Demand for US manufacturing The second shock consists of demand shifts in favor of US manufactured goods, relative to all other goods and services produced in the US. The identification of these shocks is based on a set of three restrictions. Positive demand shocks specific to US manufacturing goods should: (1) raise manufacturing output relative to aggregate output in the US; (2) raise the relative price of manufacturing in terms of other goods in the economy; and (3) raise US manufacturing output relative to Foreign manufacturing output.

Restrictions (1) and (2) now capture the theoretical prior that demand shifts move relative prices and quantities in the same direction (rather than in opposite directions, as is the case for productivity shocks); restriction (3) instead isolates shocks that are specific to US manufacturing production, relative to the other countries in our sample. These restrictions are consistent with sector-specific real demand shocks in the Mundell-Fleming-Dornbusch tradition, as well as with the good-specific taste shocks of the kind analyzed for instance by Stockman and Tesar (1995), as shown below. All other variables included in our analysis are, again, left unconstrained, so as to trace the external consequences of demand shocks in the data.

2.2 Productivity and demand shocks in a standard open-economy DSGE model

We now show that the identifying restrictions discussed above are met across a broad spectrum of open economy models. To this end, we use alternative parameterizations of a fairly general model similar to one we developed in previous work of ours (Corsetti et al. 2008b), encompassing seminal contributions such as Backus et al. (1994), Chari et al (2002) and Stockman and Tesar (1995) among others. Since the model is standard, we leave its detailed description to a web appendix. Here we briefly discuss its salient features.

The world economy consists of two countries of equal size, denoted Home and Foreign — Foreign variables are denoted with an asterisk *. Each country produces nontradable goods, denoted with \( N \) and \( N^* \), and specializes in one type of tradable goods, denoted \( T \) and \( T^* \), respectively. Tradables and nontradables are produced in a number of varieties or brands defined over a continuum of unit mass. Firms are monopolistic suppliers of one brand of goods only. These firms builds capital by investing in a composite input comprising tradables and nontradables, and combine capital with differentiated domestic labor inputs in a continuum of unit mass. Our model allows firms in the traded goods sector to set prices either in their own currency (PCP) or in the local currency of the buyer (LCP). Moreover, firms of both tradables and nontradables face a quadratic cost when adjusting their prices (this cost is however set equal to zero in steady state).
We assume a representative agent at the country level, with separable preferences over consumption, leisure and real balances. Preferences over traded and nontraded goods, and domestic and imported traded goods are modeled as CES aggregators. The international financial markets are not complete: agents only trade a noncontingent bond across borders. Finally, the central banks in both countries follow a Taylor-type rule: they set the short-term nominal interest rate as a function of the deviations of expected inflation and GDP from steady state values.

We model the two shocks of interest as follows. The Home productivity shock, $Z_t$, is a standard TFP shock to the production function of all traded good producers in the Home country. The Home demand shock is captured by a time time-varying share, $a_t$, of the traded goods in the CES aggregator of the full consumption basket (see the web appendix for details).

We proceed by fixing a small subset of parameters, such as the discount factor or the depreciation rate, and let the other parameters be drawn from uniform distributions over intervals large enough to nest many of the parameters’ estimates in the literature. The range of parameter distributions, specified in the web appendix, is kept symmetric across countries. Because of the range of parameter values considered, the model spans very different transmission mechanisms. For instance, by allowing for a low trade elasticity, the model can predict that positive productivity shocks cause a real appreciation of the terms of trade and the real exchange rate, as in Corsetti et al. (2008a,b). Moreover, since we let the price-adjustment cost parameters differ across domestic and export markets, we account for models assuming local currency pricing. The intervals for the price adjustment costs are chosen so that at one extreme prices are on average fixed for 5 quarters, while at the other extreme they are perfectly flexible — accounting for the case of producer currency pricing.

Given these intervals, we draw a large number (1 million) of parameter configurations and compute the responses to our productivity shock, $Z_t$, and to our demand shock, $a_t$. In this exercise, we are interested in documenting the effects of persistent shocks, so we set the autoregressive coefficient in the AR(1) process driving supply and demand shocks to 0.95.

As shown in Figure A1 in the web appendix, where we report the 95 percent confidence intervals for the responses of the variables restricted to identify productivity and demand shocks, the model’s responses are clearly consistent with our identification strategy. A (one percent) increase in $Z_t$ leads to (1) a fall in the relative price of locally produced traded goods ($PT/PN$), (2) an increase in Home traded output relative to Home aggregate output ($YT/Y$), (3) an increase in Home traded output relative to Foreign trade output ($YT-YT^*$),

---

15 We set the labor share in the production of traded (nontraded) goods to 39 (44) percent, based on the evidence in Stockman and Tesar (1995).

16 We use the equivalence between the law of motion for inflation in the Calvo model and in the quadratic adjustment-cost model.
and (4) an increase in US labor productivity relative to Foreign labor productivity. Similarly, a (one percent) increase in $a_t$ triggers (1) a persistent increase in the relative price of traded goods, (2) an increase in Home traded output relative to Home aggregate output, and (3) an increase in Home traded output relative to Foreign trade output.

These results are virtually unchanged whether we assume that markets are complete across borders, or that agents only trade in one uncontingent bond. Since these alternative assumptions span the widest range of international assets, we are confident that our identifying restrictions will be supported for intermediate cases.

2.3 Discussion

While we are well aware of the fact that no identification scheme is ironclad, our exercise above establishes that the proposed identification scheme is grounded in economic theory. Before proceeding with the analysis, however, it is also worth discussing specific reasons why the sign-restriction approach provides a more flexible scheme relative to the alternative of relying on exact exclusion restrictions.

Among alternative ways of identifying structural shocks, prominent instances in the literature include recursive identification (a methodology which is however unsuitable to identify demand and supply shocks), and long-run restrictions. In previous work, indeed, we have identified technology shocks to manufacturing by adapting Galí (1999) long-run restrictions to a sectoral and international context (see Corsetti et al. 2008c). Such adaptation hinges on a number of maintained assumptions, which are substantially more restrictive than those originally entertained by Galí (1999). For instance, with multiple sectors and countries, it is relatively straightforward to provide counterexamples undermining the key identifying assumption that sector-specific technology shocks are the only source of permanent movements in manufacturing labor productivity: permanent shocks in other sectors could have the same effect by inducing a reallocation of the capital stock. In addition, long-run restrictions do not easily lend themselves to the interpretation of non-technology shocks, like demand shocks. Instead, a specific advantage of the identification scheme we adopt in the present paper is that it readily allows the joint identification of demand and supply shocks.

In implementing our empirical analysis, a specific concern is that different shocks with similar effects on restricted variables may elicit opposite responses of unconstrained variables. For instance, our productivity shocks clearly capture exogenous shifts in the production function of manufacturing due to technology improvements. Yet, to the extent that measured labor productivity is endogenous, e.g. because of labor hoarding, these shocks could also correspond to other supply disturbances that do not immediately shift the production function, like embodied capital shocks or labor supply shocks. Similarly, our identified demand
shock could have domestic as well as foreign origin, so far it has disproportionate effects in favor of US manufacturing.\(^\text{17}\)

In these cases, it could be possible that our identified shocks end up having no significant estimated effects on unconstrained variables. Lack of detectable effects would clearly raise serious interpretation issues. However, to the extent that our analysis does recover some responses with a high degree of probability, our results provide model builders with useful evidence. For a candidate shock to be the main driver of our results, it should be able to account for both the estimated responses of constrained variables, e.g., an increase in labor productivity, as well as the response of any other unconstrained variables we find in the data.

### 3 The empirical framework

In our analysis, we adopt VAR methods using sign restrictions similarly to Canova and De Nicoló (2002), Faust (1998), and Uhlig (2005), for monetary policy shocks, Dedola and Neri (2007) for technology shocks, and Mountford and Uhlig (2009) for fiscal policy shocks. We go over the main elements below, starting with a discussion of the data sample and the VAR specification, and then delving into details of the identification.

In order to identify shocks which have asymmetric effects across countries, in the tradition of empirical open-economy macroeconomics, we measure all variables except trade and domestic relative prices in terms of cross-country differentials (see Clarida and Galí 1994 and Glick and Rogoff 1995). As is well understood, the alternative of expanding the empirical system to include both US and ROW variables has the clear disadvantage of running quickly against the constraint imposed by data availability, exhausting any degree of freedom in the empirical analysis.\(^\text{18}\)

Our study thus analyzes the US vis-à-vis an aggregate of the other G7 countries (Japan, Germany, the UK, Italy, Canada, and France) and three other OECD countries (Australia, Sweden, and Ireland) for which we were able to build quarterly data on hourly labor productivity in manufacturing.\(^\text{19}\) As in Section 2, we refer to this aggregate as the rest of the world

\(^{17}\)Our restrictions for the demand shock, however, rule out that an increase in manufacturing TFP in the rest of the world would qualify as a demand shock for US manufacturing, as this shock should imply a violation of restriction (3).

\(^{18}\)A potential issue in working with cross-country differentials is raised by the (implicit) assumption of symmetry across economic areas. Assuming symmetry is clearly unappealing in studies focused on small open economies, but much less consequential in our study, as we compare a large country such as the US, with a large aggregate of OECD countries. We provide evidence that our results are robust to allowing for asymmetries in the working paper version of our work (Corsetti Dedola and Leduc 2008).

\(^{19}\)These 10 countries add up to roughly half of world GDP at purchasing power parity (PPP) values, so they represent a substantial sample of the global economy. Moreover, trade flows among them also amount to over a half of their respective total trade, on average. For instance, the US trade share with the other nine countries in our sample is around 60 percent of US total trade.
(ROW). All ROW’s variables are built as an aggregate of the above mentioned countries (excluding the US), weighted according to their respective (time-varying) GDP shares at PPP values. The sample period is 1973 - 2004, covering the developments in the international monetary system after the collapse of Bretton Woods for the longest period for which we have consistent quarterly data. A detailed description of the data source is in the appendix.

We estimate several specifications of the following reduced-form VAR model (omitting the constant):

$$\mathcal{Y}_t = B(L) \mathcal{Y}_{t-1} + U_t,$$  \hspace{1cm} (1)

where the vector $\mathcal{Y}$ includes the $n$ variables of interest in levels and $B(L)$ is a lag polynomial of order $p$, which we set equal to four. The covariance matrix of the vector of reduced-form residuals $U_t$ is denoted by $\Sigma$. In our specifications (unless stated otherwise), the vector $\mathcal{Y}_t$ is 6x1. Following a common practice in open-economy VAR studies, we deal with the curse of dimensionality (due to including too many variables with relatively short samples), by keeping the first five variables in $\mathcal{Y}_t$ fixed, while changing the sixth and last variable across specifications. However, we also conduct some robustness checks estimating specifications with seven variables, without detecting any significant difference in our key results.

The first five variables in $\mathcal{Y}_t$ are as follows: (i) (the log of) quarterly labor productivity in US manufacturing, in deviation from quarterly labor productivity in manufacturing in the ROW; (ii) the (log of) US index of manufacturing production and (iii) (the log of) aggregate private consumption, both in deviation from the same variable for the ROW; (iv) (the log of) the relative US domestic producer price index over the services consumer price index; and (v) (the log of) real US manufacturing output over US real GDP.

The sixth and last variable in $\mathcal{Y}_t$ is, in turn, (the log of) real private investment in the US relative to the ROW; the ratio of US nominal net export over US nominal GDP and US real imports and exports of goods; the ratio of US external assets and liabilities and their difference over nominal GDP; and three measures of international relative prices:

$$RER_t(i) = \frac{P_t(i)}{P_t^*(i)} \quad i = CPI, PPI and Export Deflator.$$  

The price indexes $P_t(i)$ and $P_t^*(i)$ are alternatively (the log) of the CPI, PPI, and export-deflator in dollars. Note that $P_t^*(i)$ is built as a PPP, GDP-weighted aggregate of prices for the countries included in the ROW. Finally, we also look at the responses of relative equity prices in common currency and short-term interest rate differentials (in logs), where the ROW aggregates are computed as above.

---

20 We use the GDP shares as trade weights were not available for all countries going back to 1973.

21 See Hamilton (1994), chapter 20.4, for an argument in favor of this approach to model the dynamics of a vector of variables some of which may be nonstationary.
Our implementation of sign restrictions closely follows Uhlig (2005). A detailed description of the methodology is provided in the web appendix.

4 The international dimension of productivity and demand shocks to US manufacturing

In this section, we present and discuss our empirical findings for productivity and demand shocks identified using the sign restrictions discussed in Section 2. Sensitivity analysis is presented in the next section.

Operationally we require our sign restrictions to be in place for up to \( k = 20 \) quarters. In the case of the relative price of tradables (proxied by the PPI over the services CPI), we allow for possible short-run effects of nominal rigidities by having the restriction in place only from the 5th quarter (though it is immaterial for our results whether this restriction is alternatively imposed from the impact response). The choice of imposing our restrictions over a horizon of five years reflects the prior that these shocks be persistent — e.g., in line with the model results presented above. Yet, in our robustness exercises, we also experiment with restrictions over horizons of 28 and 12 quarters.

In our experiments, we consider 1000 draws from the posterior of the reduced-form coefficients, and 5000 candidate impulse response vectors. The percentage of accepted reduced form draws for which we find at least one impulse vector satisfying our restrictions is quite high, typically well above 95 percent, confirming that our posterior is not radically different from the standard unrestricted posterior (see the web appendix).

4.1 Productivity shocks

The impulse responses to a positive productivity shock are displayed in Figures 1 and 2. Each figure reports Bayesian credible intervals, by showing the 16th and 84th percentiles (the dashed lines) of the posterior distribution of the responses satisfying the restrictions detailed in Section 2.1, together with the median (the solid line). Charts for restricted variables include vertical lines, marking the horizon over which restrictions are imposed. We discuss real and financial aspects of the transmission in turns.

4.1.1 Absorption, trade and international relative prices

Figure 1 reports the response to a productivity shock of the four restricted variables (relative labor productivity and output in manufacturing, manufacturing output over GDP and the relative price of domestic manufacturing), and of eight unrestricted ones (relative con-
sumption and investment, US trade balance over GDP, US real exports and imports, two measures of US relative price of tradable goods, and the ratio of US PPI to the export deflator. These twelve variables characterize the international dimension of the US productivity shock in terms of macroeconomic and trade effects.

Starting with the four restricted variables, the median effect of the productivity shock is of the order of 0.5% and 1% for relative labor productivity and manufacturing output ($YT - YT^*$), respectively. The effect is quite persistent: notably, the 16th percentile of the productivity responses is above zero well beyond the 20 quarters over which the sign restriction is imposed. The response of (relative) manufacturing output peaks after the first year following the shock. For US manufacturing output over real GDP ($YT/Y$), the effect is also positive, but slightly smaller and less persistent in comparison. Finally, the relative price of domestic manufacturing ($PPI/CPI$ for services) displays a prolonged fall, starting already from the second quarter after the shock (this result does not change if we deflate the PPI with the aggregate CPI). This fall is well in line with the model that motivates our restrictions, implying that it is immaterial for our results whether this restriction is imposed from the fifth quarter or from the impact response.

Of the eight unrestricted variables included in Figure 1, five are related to absorption and trade, the remaining three to international relative prices. Focus first on the former group, including the (log) relative consumption ($C - C^*$) and investment ($I - I^*$), net trade scaled by nominal GDP ($NX/Y$), (log) real imports and (log) real exports. With the notable exception of real exports, the response of all these variables is also substantial and persistent: the 16th percentile remains positive for 30 quarters (in the case of investment), or longer. Relative consumption rises on impact and peaks after two years. After that, it remains positive for the entire period (10 years) shown in the figure. Relative investment also rises persistently, mimicking the response of consumption, but reacting more strongly. Median relative investment peaks at around 2 percent. The fall in net exports is more gradual: the 85th percentile of its responses falls below zero after two quarters; the median deficit reaches -0.1% of GDP after four years.

To investigate the source of the trade deterioration, we focus on the responses of real imports and real exports, and on two measures of the relative price of US tradables: one based on relative export deflators ($TOT$, for terms of trade) and the other on PPIs ($RER PPI$). Two results stand out. First, the deterioration of the trade balance is essentially driven by a rise in real imports: the response of real exports is inconclusive. Therefore, the deterioration in net trade is seemingly consistent with the notion that productivity shocks raise US absorption, as predicted by the standard intertemporal-trade approach to the current account. Note that this result questions some applied and policy literature postulating that a productivity increase in tradables should bring about an improvement in
net trade.

Second, our two measures of the relative prices of tradables increase (i.e. appreciate) persistently in the aftermath of the shock, and then undershoot the baseline over time. The probabilities that the RER PPI appreciates and the TOT improves are 80 percent or higher, for horizons of roughly five years. The similarity in the responses of these two international prices arguably reflects nominal exchange rate appreciation, in line with a vast body of evidence (see e.g. Obstfeld and Rogoff 2001).\textsuperscript{22} Furthermore, the appreciation in our measures of tradable prices is larger than the movement in the relative price PPI/CPI. This finding provides conditional evidence consistent with the analysis by Engel (1999), attributing the lion share of real exchange rate fluctuations to movements in tradable prices.\textsuperscript{23}

Finally, Figure 1 shows the response of the (log) export deflator for manufacturing goods relative to the (log) overall manufacturing PPI (\textit{EXPDEF/PPI}) for the US. A novel and intriguing finding is that the price of exported goods in domestic currency appears to fall relative to the domestic price of manufacturing: the 85th percentile of the responses of the US export deflator relative to the PPI is negative for almost 30 quarters. Vis-à-vis the real appreciation of the currency, a fall in the export deflator implies that the US export price in foreign currency adjusts by less than one-to-one with the exchange rate. This may well reflect destination-specific markup adjustment, as discussed by the vast body of evidence on deviations from the law of one price, and the recent theoretical analysis by Atkeson and Burstein (2008). A different, but possibly complementary interpretation is however suggested by recent literature in international trade and open economy macroeconomics stressing heterogeneity in productivity among firms (see, e.g., Eaton and Kortum 2002, Melitz 2003 and Ghironi and Melitz 2005). A fall in the export deflator relative to the PPI would obtain when productivity gains are stronger among exporters. According to this interpretation, our result would be driven by changes in marginal costs, rather than markup adjustment.

\textsuperscript{22}While the RER-PPI is the correct empirical counterpart of \textit{PT/PT} based on manufacturing prices, due to data limitations in countries other than the US, TOT is necessarily based on deflators for exports of both goods and services. Although the share of manufacturing in exports is preponderant, our measure of TOT is only a proxy for the relative price of exported and imported manufacturing goods.

\textsuperscript{23}Engel (1999) decomposes changes in the real exchange rate adopting the standard distinction between traded and nontraded goods, with prices \textit{PT} and \textit{PN}, so to show that under general conditions:

\[
\ln RER = \ln \frac{P}{P^*} = \ln \frac{PT}{PT^*} + \ln \frac{P/PT}{P^*/PT^*} = \ln \frac{PT}{PT^*} + (1 - \gamma) \ln \left( \frac{PN}{PT} \right) - (1 - \gamma^*) \ln \left( \frac{P^*N}{PT^*} \right),
\]

where \(\gamma\) and \(\gamma^*\) are the share of traded goods in consumption, identified with manufacturing goods. Observe that, in addition to being small relative to movements in RER-PPI, movements in \(\frac{P^*N}{PT^*}\) rate are weighted by \((1 - \gamma) < 1\) in the decomposition of the real exchange.
4.1.2 Asset prices, international portfolios, risk sharing

The response of key financial variables are shown in Figure 2. All the variables in this figure are unconstrained. The first row of the figure shows asset prices: (the log of) the CPI-based real exchange rate; the (log of the) US stock market index, relative to (the log of) an aggregate index of foreign stock markets (all in dollars); the differential between the US short-term interest rate and an aggregate of foreign short-term rates. The second row shows the market values of US foreign assets and liabilities, as estimated by Gourinchas and Rey (2007), as well as their difference, scaled by nominal GDP.24

An important result made apparent from the figure is that both asset prices and portfolios react significantly to productivity shocks. As regards asset prices, the CPI-based RER persistently appreciates in response to a positive productivity shock, again reflecting the movement of the nominal exchange rate — with a dynamic that is closely related to the other measures of US international relative prices. The US stock market increases relative to an aggregate index of foreign markets for more than five years. Observe that the relative dollar performance of the US stock market — with a differential between 2 and 7 percent — is well beyond the estimated rate of real dollar appreciation — which is below 2 percent. The response of the short-term interest rate differential is insignificant on impact, but turns positive (in favor of the US) over time, peaking 10 to 15 quarters after the shock. Comparing this response with that of the real exchange rate, a widening interest differential coexists with a stably appreciated exchange rate, suggesting (conditional) deviations from uncovered interest parity.25

The second row of Figure 2 shows that the US gross liabilities and US gross assets simultaneously rise in response to a positive shock.26 The rise in the stock of (gross) foreign assets is somewhat surprising. Since the bulk of US foreign assets are denominated in foreign currency, other things equal, the dollar appreciation documented in Figure 1 tends to reduce the value of the outstanding stock. The observed increase in the value of foreign assets must therefore be driven by some combination of capital gains, and positive purchases by US residents.

Yet, the net foreign asset position of the US overall worsens. This suggests that, conditional on productivity shocks, valuation effects (and short-term portfolio adjustment) compound, rather than offset, the effect of the trade deficit on the current account (defined as

---

24Results for the US foreign assets and liabilities are from a shorter sample, until 2004:1, dictated by the length of the series from Gourinchas and Rey (2007).
25The estimated response of interest rates is consistent with an endogenously countercyclical stance of US monetary policy, leaning against the domestic demand expansion (see Corsetti, Dedola and Leduc (2011) for a normative analysis of such a global stance of monetary policy).
26The same result is documented, following a different methodology and using a different dataset, by Corsetti and Constantinou (2012).
the difference in net foreign asset between two points in time).

These results point to novel aspects of the financial transmission mechanism. In response to productivity shocks, there is significant variation in cross-border portfolio holdings. The finding of conditional comovements of foreign assets and liabilities provide an intriguing empirical benchmark for the recent literature analyzing portfolio diversification in general equilibrium dynamic models after Devereux and Sutherland (2007) and Tille and van Wincoop (2007).

Moreover, bond and stock markets appear to provide opportunities for international diversification of risk — holdings of assets abroad could in fact insure US residents as well as residents in the rest of the world against the macroeconomic risk implied by US productivity shocks. These findings raise issues in the extent to which the existence of these hedging opportunities translates into effective international risk sharing.

A natural starting point to address this question is the well-known theoretical result (see e.g., Backus and Smith 1993 and Obstfeld and Rogoff 2001): for two national representative households residing in different countries, in a decentralized equilibrium with full consumption risk sharing the ratio of their marginal utilities should be proportional to their bilateral CPI-based real exchange rate (RER)

\[
\frac{U_{C}(.)}{U_{C^{*}}(.)} = \kappa \cdot RER,
\]

where \(C\) denotes domestic consumption, \(U_{C}(.)\) marginal utility of consumption, and \(\kappa\) is a constant reflecting the (constant) wealth distribution.\(^{27}\) Under standard assumptions about preferences, the above condition implies that domestic consumption can rise relative to foreign consumption only if its relative price is falls, i.e. the real exchange rate simultaneously depreciates (see e.g. Backus, Kehoe and Kydland (1994) in the IRBC literature, and Chari, Kehoe and McGrattan (2002) in the sticky price literature).\(^{28}\)

Our evidence is apparently at odds with the above condition for efficient risk sharing: in response to the identified productivity shock, relative consumption rises and the CPI-based RER appreciates.\(^{29}\) This finding is of crucial importance in light of the observation, often made in the literature, that a positive correlation in light of the observation, often made in the literature, that a positive correlation between relative consumption and real

---

\(^{27}\)This open-economy proposition rests on general-equilibrium theory: in an efficient allocation, across any two agents the marginal utility of consumption should be lower for the one whose consumption is cheaper.

\(^{28}\)A relevant question is whether, under complete risk sharing, a domestic relative consumption boom should be associated with a real depreciation when preferences are non-separable in consumption and leisure. However, the quantitative literature has amply shown that, conditional on technology shocks, this obtains under standard parameterizations (see e.g. Backus et al. 1994 and Corsetti et al. 2008a,b). Raffo (2011) provides a counterexample with embodied technology shocks assuming away wealth effects on the labor supply.

\(^{29}\)The joint probability of a RER appreciation and an increase in C-C*, quarter by quarter, is the same as the probability of an appreciation of US international relative prices.
appreciation in the data (as found by Backus and Smith 1993 and subsequent studies) could be reconciled with the risk-sharing condition (3) to the extent that taste shocks weaken the link between relative marginal utility and consumption. Our results document that the risk-sharing condition (3) fails to hold when measured conditional on productivity shocks only.

The finding that the cross-border diversification opportunities provided by bonds and stocks do not result into efficient consumption risk sharing resonates with the well-documented fact that countries’ portfolios are heavily biased in favor of domestic assets — recent estimates for the US (see e.g. Warnock 2011) suggests that the share of foreign equities is only up to 15 percent of total equity holding. In line with this evidence, we consider a version of our model in Section 2 in which we allow trade in Home and Foreign equities (in the form of claims to firms’ profits), in addition to trade in the noncontingent bond. When we posit a 85 percent home bias in equity portfolios, the model (in addition to being broadly consistent with our results) specifically predicts that the CPI-based real exchange rate appreciates, the relative price of domestic equities increases, but domestic consumption still rises in excess of consumption abroad, for a trade elasticity set to (the relatively low value of) 0.3 — details are reported in the web Appendix.30 In other words, under an empirically motivated calibration of the home bias in equity portfolios, the model can account for the lack of risk sharing in response to productivity shocks documented in the data. In light of the model results, however, a limited degree of equity diversification is suboptimal, raising the question of whether the high degree of home bias is due to participation costs, or to a desire to hedge against shocks other than relative productivity.

4.2 Demand shocks

Figures 3 and 4 display the impulse response functions to a positive demand shock for our benchmark specification, with the same format as the previous figures — each figure reports the 16th and 84th percentiles (the dashed lines) together with the median (the solid line) of the posterior distribution of the responses satisfying our restrictions in Section 2.1 for a demand shock. As before, vertical lines mark the horizon of restrictions.

4.2.1 Absorption, trade and international relative prices

A comparison of Figure 3 with Figure 1 emphasizes that, while the estimated effects of demand shocks on the domestic relative price of US manufacturing are quantitatively similar to those of productivity shocks, the responses of $YT - YT^*$ and $YT/Y$ are smaller and less

---

30 See Corsetti, Dedola, and Leduc (2008a,b) for details on the importance of the trade elasticity in models with incomplete asset markets.
persistent. Relative labor productivity in manufacturing (now an unrestricted variable) only rises on impact, slightly so — a natural interpretation points to short-run variations in capacity utilization corresponding to a demand-driven rise in production.

Significant effects of the shock can be detected on relative investment and international relative prices, while the credible intervals for relative consumption always include zero. In the figure, the 16th percentile of investment is positive between the 15th and the 25th quarter. US tradable prices (the PPI-based RER and \( \text{TOT} \)) mirror the pattern of investment: they do not respond in the first few quarters, but clearly appreciate after three years, around the peak of the investment response. These results confirm previous findings, that the response of different international relative prices has common determinants, in this case going beyond (and even countervailing) the pressure towards real depreciation due to the persistent decrease in the relative price of nontradable output within the US. The slight fall in the price of export goods relative to overall manufacturing, although barely significant, is again consistent with imperfect exchange rate pass-through vis-à-vis the currency appreciation (a point already discussed in relation to productivity shocks).

In light of the positive response of investment and the appreciation of international relative prices, the apparent lack of a significant response of the trade balance is surprising, especially from the perspective of the MFD framework. However, while real imports do not move appreciably, exports in real terms do tend to fall moderately in the aftermath of the exchange rate appreciation: the 85th percentile of the responses of real exports becomes negative after quarter 20.

4.2.2 Asset prices, international portfolios and risk sharing

As shown in Figure 4, demand shocks appear to have little impact on US asset prices and international portfolios. While the CPI-based RER appreciates broadly following the same pattern as the other relative prices, relative US stock prices and interest rates are barely affected. We detect a small increase in the stocks of both gross assets and liabilities. For the former, the 16th percentile of the responses rises above zero between the 5th and the 15th quarters. However, net foreign assets — once again using valuation-corrected data — remain utterly unaffected.

Overall, the evidence from our study suggests that sectoral demand shocks are not as consequential as productivity shocks for the macroeconomic process. While demand shocks that raise relative quantities and demand in the tradable sector can have in principle different origins, they do follow from fluctuations in tastes, that we use in the model analyzed in section 2 to validate our identifying assumptions. In theoretical work, taste shocks are indeed often used to capture demand shocks (one influential example is that of Stockman and Tesar.
1995). A specific contribution of our empirical analysis consists of providing evidence on a
class of shocks that are frequently posited in theoretical models and textbook analyses of
open economies; but, differently from other types of demand shocks (such as monetary and
fiscal shocks), to our knowledge never before analyzed in structural econometric work. Our
results thus allow for a useful comparison across different classes of demand disturbances.

5 Sensitivity analysis

We have analyzed the sensitivity of our results along many dimensions. First, we varied
the horizon over which sign restrictions are imposed, adding or subtracting eight quarters
relative to the baseline specification. Second, we estimated both shocks simultaneously,
imposing that they are orthogonal to each others. Third, we considered variants of our sign
restrictions and alternative specifications of the model, including addressing the symmetry
assumptions implicit in defining variable as differentials between the US and the rest of the
sample. While none of these alternative specifications has any significant impact on our
benchmark results reported above, here we focus on the results from the first two sets of
exercises.\footnote{All the results from the sensitivity analysis not reported in the paper are available upon request. Among
these, we have also estimated several 7-variable VARs, alternating marginal variables two at the
time. By way of example, we run a specification including the net trade balance and the terms of trade, another one
including real imports and exports, a third one including gross assets and liabilities and so on. None of these
exercises significantly alters our results.}

Varying the restriction horizon All our results are broadly unchanged when we re-
estimate the VAR model imposing our set of restrictions over a horizon that is shorter than
in our baseline specification. In the case of productivity shocks, for instance, the main
detectable effects of shortening the (upper) restriction horizon from 20 to 12 quarters (while
exactly replicating the analysis in all other dimensions) is that the response of all variables
to the productivity shock becomes somewhat less persistent, as we now allow for a number
of identified shocks which affect variables for a shorter period of time and thus affect the
posterior distribution of impulse responses.

By the same token, imposing that our restrictions bind for up to 28 quarters, i.e., eight
quarters more than our baseline case, increases the persistence of the estimated effects of
productivity and demand shocks, as expected. However, lengthening the restriction horizon
makes it more difficult to find productivity shocks in the data — we end up rejecting a larger
fraction of draws from the reduced-form posterior of our VAR. Similar considerations apply
to the case of demand shocks. To save space, we do not report figures for these exercises.
Orthogonal productivity and demand shocks  In our analysis, we require the two shocks to have opposite effects on the U.S. relative price of manufacturing vis-à-vis CPI services, while always increasing U.S. manufacturing output relative to real GDP. In this respect, our identification strategy limits the risk that our results confound their effects. Yet, as we identify productivity and demand shocks individually, without requiring both to be present in the data and be orthogonal to each other, our estimated effects of either shock could potentially be biased — for instance, if the two shocks happen to be negatively correlated with each other. Specifically, in the case of the demand shock, it may happen that some of the estimated responses, while picking up the assumed positive demand shock, could be contaminated by a (weaker) negative productivity shock. This could have the effect of strengthening the positive response of the domestic relative price of manufacturing, while attenuating that of other variables, like relative output or consumption.

Moreover, orthogonality may have a further important consequence for our results. Since the number of restrictions imposed simultaneously is larger, it may be more difficult to find the two shocks in the data, leading us to reject a much higher number of reduced forms in our estimation procedure. This could affect our finding, that each shock individually is very likely to be present in the data.

However, when we estimate orthogonal productivity and demand shocks imposing the restrictions simultaneously (see the web Appendix for details on the methodology), we find virtually no difference from the results for each shock in isolation presented in the previous section, nor any drop in the acceptance rate of reduced form draws. Once again, to save space, we do not report figures for these exercises.

6 Conclusions

In this paper, we have produced empirical evidence on the international dimension of productivity and demand shocks in the U.S., vis-à-vis an aggregate of other large industrial countries. Using a novel identification scheme and methodology, as well as a broad range of variables, we have analyzed real and financial channels through which shocks propagate internationally, presenting a comprehensive empirical framework for assessing alternative models of the transmission mechanism. In our findings, sectoral real demand shocks in US manufacturing have limited aggregate effects on absorption and net trade, yet they affect specific demand components, especially investment, and appreciate the real value of the dollar. Conversely, sectoral productivity shocks in US manufacturing have an unbalanced impact on aggregate demand across countries, raising domestic absorption and the trade deficit, while also strengthening the international prices of US goods as well as the US real exchange rate.
According to our results, real demand shocks do not produce the external crowding out effects emphasized by the Mundell-Fleming model. Positive productivity shocks raising the supply of US manufacturing goods do not appear to be matched by a fall in their international prices indicating that, at least over the business cycle, the movements in the relative prices of US manufacturing goods are far from providing risk insurance opportunities against productivity fluctuations in the sector. Interestingly, we find that these opportunities appear to be provided by widely traded assets (especially stocks), whose prices are quite responsive to our shocks. While investing in international assets can in principle help hedging against US productivity risk, however, we detect little aggregate evidence of efficient diversification.

Our structural analysis provides empirical guidance for further theoretical and empirical investigations in a number of dimensions. A key issue concerns the extent to which this conditional evidence could be reconciled with recent dynamic models featuring endogenous portfolio diversification, as well as with quantitative models allowing for goods quality and/or variety adjustment over the business cycle. More generally, our findings suggest that asymmetric, unbalanced fluctuations in aggregate demand across countries — hard to match by models postulating frictionless financial markets — should be reconsidered as an essential element in the analytical characterization of the international transmission mechanism.

References


Figure 1: Macroeconomic and Trade Responses to a Positive Home Productivity Shock
Figure 2: Asset Prices, International Portfolios and Risk Sharing -- Home Productivity Shock
Figure 3: Macroeconomic and Trade Responses to a Positive Home Demand Shock

- Labor Productivity
- YT-YY*
- PPI/CPI
- YT/Y
- C-C*
- I-I*
- NX/Y
- Real Imports
- Real Exports
- RER PPI
- TOT
- EXPDEF/PPI

The graphs depict the responses of various economic indicators to a positive home demand shock, showing changes over time in quarters.
Figure 4: Asset Prices, International Portfolios and Risk Sharing -- Home Demand Shock

- RER CPI
- Foreign Assets over GDP
- Foreign Liabilities over GDP
- Relative Stock Market Prices
- Net Foreign Assets over GDP
Appendix A  Model setting (Web appendix)

Appendix A.1 The Household’s Problem

Preferences  The representative Home agent in the model maximizes the expected value of her lifetime utility, where the instantaneous utility is given by

\[
\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{\left( \frac{M_{t+1}}{P_t} \right)^{1-\sigma}}{1-\sigma} + \alpha \frac{(1-L_t)^{1-\sigma}}{1-\sigma},
\]

where instantaneous utility \( U \) is a function of a consumption index, \( C_t \), leisure, \((1-L_t)\), and real money balances \( \frac{M_{t+1}}{P_t} \). We set \( \chi \) arbitrarily to a small value (0.1) and calibrate \( \alpha \) so that the representative agent spends a third of his time working.

Households consume all types of (domestically-produced) nontraded goods, \( N \) (which come in individual brands \( n \)) and both types of traded goods \( T \) and \( T^* \) (which come in individual brands \( h \) and \( f \), respectively). So \( C_t(n,j) \) is consumption of brand \( n \) of Home nontraded good by agent \( j \) at time \( t \); \( C_t(h,j) \) and \( C_t(f,j) \) are the same agent’s consumption of Home brand \( h \) and Foreign brand \( f \) of tradable goods. For each type of good, we assume that one brand is an imperfect substitute for all other brands, with constant elasticity of substitution \( \theta_T \) and \( \theta_N > 1 \). Consumption of Home and Foreign goods by Home agent \( j \) is defined as:

\[
C_{T,t}(j) \equiv \left[ \int_0^1 C_t(h,j)^{\theta_T^{-1}} \, dh \right]^{\frac{1}{\theta_T^{-1}}}, \quad C_{T^*,t}(j) \equiv \left[ \int_0^1 C_t(f,j)^{\theta_T^{-1}} \, df \right]^{\frac{1}{\theta_T^{-1}}}, \quad (5)
\]

\[
C_{N,t}(j) \equiv \left[ \int_0^1 C_t(n,j)^{\theta_N^{-1}} \, dn \right]^{\frac{1}{\theta_N^{-1}}}.
\]

The full consumption basket, \( C_t \), in each country is defined by the following CES aggregator

\[
C_t \equiv \left[ a_t^{-1-\phi} \left[ C(C_{T,t},C_{T^*,t}) \right]^{\phi} + (1-a_t)^{1-\phi} C_{N,t} \right]^{\frac{1}{\phi}}, \quad \phi < 1,
\]

where \( a_t \) dictates a time-varying shock on the consumption of traded goods assumed that it follows an AR(1) process, with a steady value \( \bar{a} \). Movements in \( a_t \) will capture our demand shock. The constant elasticity of substitution between \( C_{N,t} \) and aggregate traded consumption \( C(C_{T,t},C_{T^*,t}) \) is given by \( \frac{1}{1-\phi} \).

Similarly, the consumption index of traded goods \( C_{T,t} \) is given by the following CES aggregator

\[
C \left( C_T, C_{T^*} \right) = \left[ a_T^{-1-\rho} C_T^{\rho} + (1-a_T)^{1-\rho} C_{T^*}^{\rho} \right]^{\frac{1}{\rho}}, \quad \rho < 1.
\]

\( ^{32} \)We use an endogenous discount factor to guarantee the existence of a unique invariant distribution of wealth, independent of initial conditions, which will allow us to use standard numerical techniques to solve the model around a stable nonstochastic steady state when only a non-contingent bond is traded internationally (see Corsetti, Dedola, and Leduc [2008b]).
Budget constraints and asset markets  Home and Foreign agents trade an international bond, $B$, which pays in units of Home currency and is zero in net supply. Households derive income from working, $W_t L_t$, from renting capital to firms, $R_t K_t$, from previously accumulated units of currency, and from the proceeds from holding the international bond, $(1 + i_t) B_t$, where $i_t$ is the nominal bond’s yield, paid at the beginning of period $t$ in domestic currency but known at time $t - 1$. Households pay non-distortionary (lump-sum) net taxes $T_t$ denominated in Home currency, and use their disposable income to consume, invest in domestic capital, and buy bonds $B_{t+1}$. Only Home residents hold the Home currency, $M_t$. The individual flow budget constraint for the representative agent $j$ in the Home country is therefore:

$$M_t(j) + B_{t+1}(j) \leq M_{t-1}(j) + (1 + i_t) B_t(j) + R_t K_t(j)$$

$$+ \int_0^1 \Pi(h, j) dh + \int_0^1 \Pi(n, j) dn + W_t L_t(j) - T_t(j) - P_{T,t}C_{T,t}(j) - P_{T^*,t}C_{T^*,t}(j) - P_{N,t}C_{N,t}(j) - P_{inv,t}I_t(j)$$

where $\int \Pi(h, j) dh + \int \Pi(n, j) dn$ is the agent’s share of profits from all firms $h$ and $n$ in the economy. The price indexes as as follows: $P_{T,t}$ denote the price of the Home traded good, $P_{T^*,t}$ is the consumer price of Home imports; $P_{N,t}$ is the price of nontraded goods; and $P_t$ is the consumer price index, and $P_{inv,t}$ is the price of investment.

Aggregate investment is a Cobb-Douglas composite of tradable and nontradable goods, in line with the evidence in Bems (2005), with the tradable component of aggregate investment is obtained through the same CES aggregator as that of tradable consumption. This way we introduce in the model the notion of intermediate imported inputs that contribute to the formation of capital in the economy. The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = I_t + (1 - \delta) K_t + \frac{b}{2} \left( \frac{I_t}{K_t} - \delta \right) ^2,$$

where $b$ is an adjustment cost parameter.

The household’s problem then consists of maximizing lifetime utility, defined by (4), subject to the constraints (8) and (9).

Appendix A.2  Firms’ problem

Firms producing Home tradables (T) and Home nontradables (N) are monopolist in their variety of good; they employ a technology that combines domestic labor and capital inputs, according to the following Cobb-Douglas functions:

$$Y_t(h) = Z_t(h) K_t(h)^{1-\xi} L_t(h)^{\xi}$$

$$Y_t(n) = K_t(n)^{1-\xi} L_t(n)^{\xi},$$

where $Z_t(h)$ is a supply shock to the traded goods sector assumed to follow an AR(1) process. We assume that capital and labor are freely mobile across sectors.

A.2
Our model allows firms in the traded goods sector to set prices in local currency. The Home firm can therefore set domestic prices, \( p_t(h) \), as well as foreign prices, \( p^*_t(h) \), both in local currencies. Similarly, firms in the nontraded goods sector choose \( p_t(n) \).

Moreover, firms in both sectors face a quadratic cost when adjusting their prices (this cost is however set equal to zero in steady state). To change its product prices, a firm needs to consume a CES aggregated basket of all the goods in the same sector of the economy; thus the price-adjustment costs faced by firms in the traded and non-traded goods sector are respectively:

\[
AC_{pT,t}^T(h) = \frac{\kappa_T^p}{2} \left( \frac{p_t(h)}{p_{t-1}(h)} - \pi \right)^2 D_{T,t}, \quad AC_{pT,t}^n(h) = \frac{\kappa_T^p}{2} \left( \frac{p^*_t(h)}{p^*_{t-1}(h)} - \pi \right)^2 D_{T,t},
\]

and

\[
AC_{pT,t}^n(n) = \frac{\kappa_N^p}{2} \left( \frac{p_t(n)}{p_{t-1}(n)} - \pi \right)^2 D_{N,t}.
\]

Note that our model also encompasses producer currency pricing when prices set in the foreign market are flexible.

It is worth recalling that there is an equivalence between the law of motion for inflation in the Calvo model and in the quadratic adjustment-cost model. A typical Calvo price-setting model implies a (log-linearized) stochastic difference equation for inflation of the form

\[
\pi_t = \beta E_{t} \pi_{t+1} + \lambda m c_t,
\]

where \( m c_t \) is the firm’s real marginal cost of production, and \( \lambda = \frac{(1-q)(1-\beta)}{q} \), with \( q \) being the constant probability that a firm must keep its price unchanged in any given period and \( \beta \) the subjective discount factor (see Galí and Gertler 1999). The quadratic adjustment-cost model gives a similar (log-linearized) difference equation for inflation, but with \( \tilde{\lambda} = \frac{\theta_j - 1}{\kappa_j^p \pi^2} \), \( J=T,N \). A value of \( q \) and \( \beta \) determine \( \tilde{\lambda} \), which we translate into a value for \( \kappa^p \) for a given \( \theta_j \).

### Appendix A.3 Monetary Policy

We assume that central banks follow a Taylor-type rule setting the short-term nominal interest rate as a function of the deviations of expected inflation and GDP from steady state values:

\[
R_t = \rho_R R_{t-1} + \beta_\pi (1-\rho_R)(\pi_t - \pi^*) + \beta_y (1-\rho_R)(y_t - y^*).
\]
Appendix A.4 Extension: International Equity Portfolios

We consider an extension of our model that allows for trade in international equities of firms in the traded-goods sector. Let $s_t(h)$ and $s_t(f)$ be the fraction of Home firm $h$'s and Foreign firm $f$'s equities respectively held by the Home household and denote by $Q_t(h)$ and $Q_t(f)$ the price of those equities. So, the Home household’s budget constraint is modified as follows

$$M_t(j) + B_{t+1}(j) + \int Q_t(h)s_t(h,j) + \int \varepsilon s_t(f,j) \leq M_{t-1}(j) + (1 + \iota_t)B_t(j) + R_tK_t(j)$$

(14)

$$+ \int (Q_t(h) + d_t(h))s_{t-1}(h,j) + \int \varepsilon_t(Q_t(f) + d_t(f))s_{t-1}(f,j)$$

(15)

$$W_tL_t(j) - T_t(j) - P_{T,t}C_{T,t}(j) - P_{T*,t}C_{T*,t}(j) - P_{N,t}C_{N,t}(j) - P_{inv,t}I_t(j)$$

where $d_t(h)$ and $d_t(f)$ are Home and Foreign firms dividends and where $\varepsilon_t$ represents the nominal exchange rate.

For the calibration of this version of the model, we keep the values of the discount factor, the labor share in traded (nontraded) goods production, and the capital depreciation rate to those in Table A1. We calibrate the remaining parameter values as follows. We set the weight on traded goods, $a_T$, to generate a 10 percent import share in steady state. Similarly, the steady State weight on Home traded goods, $\bar{a}$, so that the consumption of nontraded goods make up 50 percent of total consumption. We assume markups in all sector of 20 percent and that prices change on average every four quarters. We set the traded-non traded goods’ elasticity of substitution to 0.74 and the trade elasticity to 0.3. The parameters of the interest-rate rule are set as follows: $\rho_R = 0.84$, $\beta_R = 2.19$, and $\beta_y = 0.3$. Finally, we set the investment adjustment cost to 50, the mid range of our interval in Table A1.

Under this calibration of the model and postulating an 85 percent home bias in equity portfolios, we find that a productivity shock leads to an appreciation of the currency in real terms and a rise in Home consumption relative to consumption abroad, as shown in Figure A2.
Table A1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Labor share in traded goods production, $\xi$</td>
<td>0.39</td>
</tr>
<tr>
<td>Labor share in nontraded goods production, $\zeta$</td>
<td>0.44</td>
</tr>
<tr>
<td>Capital depreciation rate, $\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Weight on traded goods, $a_T$</td>
<td>[0.75, 0.95]</td>
</tr>
<tr>
<td>Steady State weight on Home traded goods, $\bar{\pi}$</td>
<td>[0.4, 0.6]</td>
</tr>
<tr>
<td>Nontraded and traded goods steady state markup, $\frac{\theta_N}{\theta_{N-1}} = \frac{\theta_T}{\theta_{T-1}}$</td>
<td>[1.1, 1.2]</td>
</tr>
<tr>
<td>Price adjustment costs, $\kappa^p_N, \kappa^p_T, \kappa^{*p}_T$</td>
<td>[0, 187]</td>
</tr>
<tr>
<td>Trade elasticity, $\frac{1}{1 - \rho}$</td>
<td>[0.5, 5]</td>
</tr>
<tr>
<td>Traded-non traded goods’ elasticity of substitution, $\frac{1}{1 - \phi}$</td>
<td>[0.5, 1.5]</td>
</tr>
<tr>
<td>Interest-rate smoothing, $\rho_R$</td>
<td>[0.25, 0.99]</td>
</tr>
<tr>
<td>Coefficient on inflation, $\beta_\pi$</td>
<td>[1.05, 3]</td>
</tr>
<tr>
<td>Coefficient on output, $\beta_y$</td>
<td>[0, 0.7]</td>
</tr>
<tr>
<td>Investment adjustment cost, $b$</td>
<td>[0, 100]</td>
</tr>
</tbody>
</table>
Appendix B  Data description and sources (Web appendix)

United States

Labor productivity: Index of output per hour of all persons in manufacturing sector, seasonally adjusted, 1992 = 100 (Bank of International Settlements and Dept. of Labor).

Manufacturing output: Index of industrial production in manufacturing, seasonally adjusted, 2000 = 100 (Federal Reserve Board)

Consumption: Private final consumption expenditure, volume in national currency, seasonally adjusted (OECD, Economic Outlook Database).

Nominal GDP: Gross domestic product, value, market prices in national currency, seasonally adjusted (OECD, Economic Outlook Database)

Net exports: Nominal net exports of goods & services, value in national currency, seasonally adjusted (OECD, Economic Outlook Database)

Real imports and exports: Real imports and exports of goods, national currency, seasonally adjusted, 2000 = 100 (NIPA, Table 4.2.3)

PPI index: Producer price index of manufactured products, seasonally adjusted, 2000 = 100 (OECD, Main Economic Indicators Database)

CPI total: Consumer price index all items, seasonally adjusted, 2000 = 100 (OECD, Main Economic Indicators Database)

CPI services: Consumer price index for services less energy services, seasonally adjusted; 1982-84 = 100, monthly converted to quarterly averages (BLS)

Export deflator: Exports of goods, deflator, seasonally adjusted, national accounts basis; 2000 = 100 (OECD, Economic Outlook Database)

Short-term rate: Federal Fund Rate, quarterly (IMF, International Financial Statistics)


US Foreign assets and liabilities: Valuation adjusted US net foreign assets, gross foreign assets and gross foreign liabilities (Gourinchas and Rey (2007), Appendix B)

CPI-based real exchange rate: Index of ratio of US CPI (total) to aggregate CPI (total) of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1970q1 = 100 (authors calculations based on OECD, Economic Outlook Database)

PPI-based real exchange rate: Index of ratio of US PPI (manufacturing) to aggregate PPI (manufacturing) of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1971q1 = 100 (authors calculations based on OECD, Economic Outlook Database)
Terms of trade: Index of ratio of US export deflator (goods and services) to aggregate export deflator (goods and services) of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1970q1 = 100 (authors calculations based on OECD, Economic Outlook Database)

Relative stock prices: Index of ratio of US stock prices to aggregate stock prices of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1970q1 = 100 (authors calculations based on IMF, International Financial Statistics)

Rest of the world

The rest of the world comprises Japan, Germany, UK, Italy, France, Canada, Australia, Sweden and Ireland. This choice was dictated by data availability regarding hourly productivity in manufacturing.

Individual country’s variables were aggregated, first by taking quarterly growth rates to remove national basis effects; then cross-country average growth rates were computed with weights based on each country’s GDP share in the 9-country aggregate calculated at annual purchasing power parity (PPP) values. Average growth rates were then cumulated starting from the initial base year to obtain levels.

Annual PPP-based GDP shares are from the IMF’s World Economic Outlook Database from 1980; before 1980 they were computed directly on the basis of annual GDP at PPP values from the OECD’s Economic Outlook Database.

Labor productivity: Aggregate of country-specific indexes of output per hour of all persons in manufacturing sector, seasonally adjusted, 1970q1 = 100 (authors calculations based on national statistical sources, BIS and IMF)

Manufacturing output: Aggregate of country-specific indexes of industrial production, manufacturing, seasonally adjusted, 1970q1 = 100 (authors calculations based on national statistical sources, BIS and IMF)

Consumption and investment: Aggregate of country-specific private final consumption expenditure, volumes in national currency, seasonally adjusted, 1970q1 = 100 (authors calculations based on OECD, Economic Outlook Database).

Stock prices: Aggregate of country-specific share prices, 1970q1 = 100 (authors calculations based on IMF, International Financial Statistics and BIS (for UK only)).

Appendix C  Methodology (Web appendix)

In the structural VAR literature, identification amounts to providing enough restrictions as to solve uniquely for the following decomposition of the $n \times n$ estimated covariance matrix of the reduced-form VAR residuals $\Sigma$ (up to an orthonormal transformation $Q$ such that $QQ^\top = I$):

$$\Sigma = A_0 A_0^\top.$$ 

This matrix equation defines a one-to-one mapping from the vector of orthogonal structural shocks $V$ to the reduced form residuals $U$, $U = A_0 V$. Because of the orthogonality assumption, and the symmetry of $\Sigma$, at least $\frac{n(n-1)}{2}$ restrictions on $A_0$ need to be imposed.

The $j$-th column of the identification matrix $A_0$, $A_{0,j}$, is called an impulse vector in $\mathbb{R}^n$, as it maps the innovation to the $j$-th structural shock $v_j$ into the contemporaneous impact responses of all the $n$ variables, $\Psi_{0,j}$. With the structural impulse vector $A_{0,j}$ in hand, the set of all structural impulse responses of the $n$ variables up to the horizon $k$, $\Psi_{1,j}, ..., \Psi_{k,j}$ can then be computed using the estimated coefficient matrix $B(L)$ of the reduced-form VAR, $B_1, B_2, ... B_p$:

$$\Psi_{s,j} = \sum_{h=0}^{s} B_{s-h} \Psi_{h,j}, \quad s \geq 1, B_{s-h} = 0, s - h \geq p;$$

$$\Psi_{0,j} = A_{0,j}.$$ 

Proposition 1 in Uhlig (2005) shows that any structural impulse vector $A_{0,j}$ arising from a given identifying matrix $A_0$ can be represented as $Pq$, for an appropriate vector $q$ belonging to the hypersphere of unitary radius $S^n \subset \mathbb{R}^n$, and an arbitrary matrix $P$ such that $PP^\top = \Sigma$. For instance, natural candidates for the orthogonal decomposition $P$ are either the eigenvalue-eigenvector or the Cholesky decomposition of $\Sigma$.

Our procedure to obtain estimates of impulse responses consistent with a given set of assumed sign restrictions can be described as follows. Economic theory can be brought to bear, as in Uhlig (2005) or Dedola and Neri (2007), to attribute all the probability mass to the event that the responses of $m \leq n$ variables (e.g., relative labor productivity, relative output, and so on) to the specific structural shock of interest have a given (positive or negative) sign for $s \leq k$ quarters. For instance, Uhlig (2005) appeals to standard monetary theory and assumes that a contractionary monetary policy shock in the US uniquely brings about a hike in the federal funds rate, a drop in the price level and a contraction in money demand (nonborrowed reserves).

Thus, on the basis of theoretical insights, a-priori a non-zero probability is attributed only to structural impulse vectors $A_{0,j}$ which (for a given reduced-form estimate of the VAR) yield impulse responses $\Psi_{s,j}$ whose signs are consistent with the assumed restrictions. It is important to stress that sign restrictions allow us to implement natural theoretical assumptions in the identification procedure (e.g., supply-side shocks should move relative quantities and prices in the opposite direction). However they also lead to a plurality of candidate structural impulse responses. Rather than as a shortcoming, this is a potentially important advantage relative to exact exclusion restrictions, such as long-run restrictions,
that could be sensitive to small perturbations to model specification and parameterization.\textsuperscript{33}

As argued by Uhlig (2005), the fact that the Bayesian approach views the VAR parameters as random variables makes it particularly suited to interpreting and implementing sign restrictions. From a Bayesian point of view, on the one hand, the approach amounts to attributing zero probability to reduced-form parameter realizations for which impulse responses contravene the assumed set of sign restrictions. On the other hand, all the impulse responses from the same reduced-form realization that satisfy those restrictions are attributed the same probability a priori. Thus, one can use standard Bayesian methods for estimation and inference, to obtain measures of the uncertainty about estimated impulse responses.

Formally, (i) under a standard diffuse prior on the VAR reduced-form parameters $B(L)$ and $\Sigma$, (ii) under a uniform prior on rotation vectors $q$ in $S^n$ yielding impulse responses in accord with restrictions, and (iii) assuming a Gaussian likelihood for the data sample at hand, the posterior density of the reduced-form VAR parameters with the type of restrictions we implement will be proportional to a standard Normal-Wishart $-\text{ whose parameters are known functions of the OLS-MLE estimates of the VAR reduced form (with the proportionality factor being an indicator function equal to one when at least one rotation } q \text{ exists yielding impulse responses consistent with the restrictions).}$ This implies that all impulse responses satisfying the restrictions obtained from a given reduced form draw are attributed the same posterior probability, according to the standard Normal-Wishart distribution.

Therefore, it is possible to simulate the posterior distribution of impulse responses consistent with our sign restrictions by jointly drawing from the Normal-Wishart posterior for $\Sigma$, $B(L)$ and the uniform for $q$ over $S^n$, discarding the impulse responses that violate the restrictions. This could be operationalized by using the following algorithm suggested by Uhlig (2005): for a given estimate of the VAR reduced-form matrices $\Sigma$ and $B(L)$, and the associated decomposition $P$, we draw (a large number of) candidate $q$ vectors from a uniform distribution over $S^n$, and compute the associated impulse vector $A_{0,j}$ and impulse response matrix $\Psi$, discarding those that do not satisfy the assumed sign restrictions. In practice, the $q$ vectors are drawn from a multivariate standard normal and normalized with their Euclidean norm to make sure they have unitary length.

To produce two sets of candidate impulse responses which are orthogonal on impact, the required change in the procedure described above is straightforward. We now need to find two vectors, $q_1$ and $q_2$, both belonging to the hypersphere of unitary radius $S^n \subset \mathbb{R}^n$, which also satisfy the additional orthogonality condition $q_1^T q_2 = 0$; then, we can compute the two impulse vectors $Pq_1$ and $Pq_2$ and the related impulse responses, verifying that they satisfy the sign restrictions for productivity and demand shocks. In practice, the vectors are again drawn from a multivariate standard normal, then orthogonalized and normalized with their Euclidean norm to make sure they have unitary length.

\textsuperscript{33}For instance, to adapt long-run restrictions to identify manufacturing shocks in an international context, Corsetti et al (2008c) had to assume that productivity shocks are exactly nonstationary and the only source of a stochastic trend in both the level of labor productivity, and its differential across countries. This amounts to assume permanent differences in international productivity levels. Most importantly, in contrast with the kind of sign-restrictions we use in this paper, long-run restrictions require heavy reliance on unit root tests to buttress key specification assumptions in all the variables included in the VARs.
Appendix A  Model setting (Web appendix)

Appendix A.1 The Household’s Problem

Preferences  The representative Home agent in the model maximizes the expected value of her lifetime utility, where the instantaneous utility is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \left( \frac{M_{t+1}}{P_t} \right)^{1-\sigma} + \alpha \frac{(1-L_t)^{1-\sigma}}{1-\sigma}, \quad (4)$$

where instantaneous utility $U$ is a function of a consumption index, $C_t$, leisure, $(1-L_t)$, and real money balances $\frac{M_{t+1}}{P_t}$. We set $\chi$ arbitrarily to a small value (0.1) and calibrate $\alpha$ so that the representative agent spends a third of his time working.

Households consume all types of (domestically-produced) nontraded goods, $N$ (which come in individual brands $n$) and both types of traded goods $T$ and $T^*$ (which come in individual brands $h$ and $f$, respectively). So $C_t(n,j)$ is consumption of brand $n$ of Home nontraded good by agent $j$ at time $t$; $C_t(h,j)$ and $C_t(f,j)$ are the same agent’s consumption of Home brand $h$ and Foreign brand $f$ of tradable goods. For each type of good, we assume that one brand is an imperfect substitute for all other brands, with constant elasticity of substitution $\theta_T$ and $\theta_N > 1$. Consumption of Home and Foreign goods by Home agent $j$ is defined as:

$$C_{T,t}(j) \equiv \left[ \int_0^1 C_t(h,j) \frac{\theta_T - 1}{\theta_T} dh \right]^{\theta_T}, \quad C_{T^*,t}(j) \equiv \left[ \int_0^1 C_t(f,j) \frac{\theta_{T^*} - 1}{\theta_{T^*}} df \right]^{\theta_{T^*}},$$

$$C_{N,t}(j) \equiv \left[ \int_0^1 C_t(n,j) \frac{\theta_N - 1}{\theta_N} dn \right]^{\theta_N}. \quad (5)$$

The full consumption basket, $C_t$, in each country is defined by the following CES aggregator

$$C_t \equiv \left[ a_t^{1-\phi} [C(C_{T,t}, C_{T^*,t})]^{\phi} + (1-a_t)^{1-\phi} C_{N,t}^{\phi} \right]^{\frac{1}{\phi}}, \quad \phi < 1, \quad (6)$$

where $a_t$ dictates a time-varying shock on the consumption of traded goods assumed that it follows an AR(1) process, with a steady value $\bar{a}$. Movements in $a_t$ will capture our demand shock. The constant elasticity of substitution between $C_{N,t}$ and aggregate traded consumption $C(C_{T,t}, C_{T^*,t})$ is given by $\frac{1}{1-\phi}$.

Similarly, the consumption index of traded goods $C_{T,t}$ is given by the following CES aggregator

$$C(C_{T}, C_{T^*}) = \left[ a_T^{1-\rho} C_T^{\rho} + (1-a_T)^{1-\rho} C_{T^*}^{\rho} \right]^{\frac{1}{\rho}}, \quad \rho < 1. \quad (7)$$

\[31\] We use an endogenous discount factor to guarantee the existence of a unique invariant distribution of wealth, independent of initial conditions, which will allow us to use standard numerical techniques to solve the model around a stable nonstochastic steady state when only a non-contingent bond is traded internationally (see Corsetti, Dedola, and Leduc [2008b]).
Budget constraints and asset markets  Home and Foreign agents trade an international bond, $B$, which pays in units of Home currency and is zero in net supply. Households derive income from working, $W_t L_t$, from renting capital to firms, $R_t K_t$, from previously accumulated units of currency, and from the proceeds from holding the international bond, $(1 + i_t)B_t$, where $i_t$ is the nominal bond’s yield, paid at the beginning of period $t$ in domestic currency but known at time $t-1$. Households pay non-distortionary (lump-sum) net taxes $T_t$, denominated in Home currency, and use their disposable income to consume, invest in domestic capital, and buy bonds $B_{t+1}$. Only Home residents hold the Home currency, $M_t$.

The individual flow budget constraint for the representative agent $j$ in the Home country is therefore:

$$M_t(j) + B_{t+1}(j) \leq M_{t-1}(j) + (1 + i_t)B_t(j) + R_t K_t(j) + \int_0^1 \Pi(h,j) dh + \int_0^1 \Pi(n,j) dn + W_t L_t(j) - T_t(j) - P_{T,t} C_{T,t}(j) - P_{T^*,t} C_{T^*,t}(j) - P_{N,t} C_{N,t}(j) - P_{\text{INV},t} I_t(j)$$

(8)

where $\int \Pi(h,j) dh + \int \Pi(n,j) dn$ is the agent’s share of profits from all firms $h$ and $n$ in the economy. The price indexes as as follows: $P_{T,t}$ denote the price of the Home traded good, $P_{T^*,t}$ is the consumer price of Home imports; $P_{N,t}$ is the price of nontradable goods; and $P_t$ is the consumer price index, and $P_{\text{INV},t}$ is the price of investment.

Aggregate investment is a Cobb-Douglas composite of tradable and nontradable goods, in line with the evidence in Bems (2005), with the tradable component of aggregate investment is obtained through the same CES aggregator as that of tradable consumption. This way we introduce in the model the notion of intermediate imported inputs that contribute to the formation of capital in the economy. The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = I_t + (1 - \delta)K_t + \frac{b}{2} \left( \frac{I_t}{K_t} - \delta \right)^2,$$

(9)

where $b$ is an adjustment cost parameter.

The household’s problem then consists of maximizing lifetime utility, defined by (4), subject to the constraints (8) and (9).

Appendix A.2  Firms’ problem

Firms producing Home tradables (T) and Home nontradables (N) are monopolist in their variety of good; they employ a technology that combines domestic labor and capital inputs, according to the following Cobb-Douglas functions:

$$Y_t(h) = Z_t(h) K_t(h)^{1-\xi} L_t(h)^\xi$$

$$Y_t(n) = K_t(n)^{1-\xi} L_t(n)^\xi,$$

(10)

where $Z_t(h)$ is a supply shock to the traded goods sector assumed to follow an AR(1) process. We assume that capital and labor are freely mobile across sectors.
Our model allows firms in the traded goods sector to set prices in local currency. The Home firm can therefore set domestic prices, \( p_t(h) \), as well as foreign prices, \( p_t^*(h) \), both in local currencies. Similarly, firms in the nontraded goods sector choose \( p_t(n) \). Moreover, firms in both sectors face a quadratic cost when adjusting their prices (this cost is however set equal to zero in steady state). To change its product prices, a firm needs to consume a CES aggregated basket of all the goods in the same sector of the economy; thus the price-adjustment costs faced by firms in the traded and non-traded goods sector are respectively:

\[
AC_{T,t}^p(h) = \frac{\kappa_T^p}{2} \left( \frac{p_t(h)}{p_{t-1}(h)} - \pi \right)^2 D_{T,t}, \quad AC_{n,t}^p(n) = \frac{\kappa_N^p}{2} \left( \frac{p_t(n)}{p_{t-1}(n)} - \pi \right)^2 D_{N,t}.
\]

and

\[
AC_{T,t}^p(h) = \frac{\kappa_T^p}{2} \left( \frac{p_t^*(h)}{p_{t-1}^*(h)} - \pi \right)^2 D_{T,t}, \quad AC_{n,t}^p(n) = \frac{\kappa_N^p}{2} \left( \frac{p_t^*(n)}{p_{t-1}^*(n)} - \pi \right)^2 D_{N,t}.
\]

Note that our model also encompasses producer currency pricing when prices set in the foreign market are flexible.

It is worth recalling that there is an equivalence between the law of motion for inflation in the Calvo model and in the quadratic adjustment-cost model. A typical Calvo price-setting model implies a (log-linearized) stochastic difference equation for inflation of the form \( \pi_t = \beta E_t \pi_{t+1} + \lambda mc_t \), where \( mc_t \) is the firm’s real marginal cost of production, and \( \lambda = \frac{(1-q)(1-\beta q)}{q} \), with \( q \) being the constant probability that a firm must keep its price unchanged in any given period and \( \beta \) the subjective discount factor (see Galí and Gertler 1999). The quadratic adjustment-cost model gives a similar (log-linearized) difference equation for inflation, but with \( \tilde{\lambda} = \frac{\theta_3 - 1}{\kappa_3^p \pi^2} \), \( J = T, N \). A value of \( q \) and \( \beta \) determine \( \tilde{\lambda} \), which we translate into a value for \( \kappa^p \) for a given \( \theta_3 \).

**Appendix A.3 Monetary Policy**

We assume that central banks follow a Taylor-type rule setting the short-term nominal interest rate as a function of the deviations of expected inflation and GDP from steady state values:

\[
R_t = \rho_R R_{t-1} + \beta_\pi (1 - \rho_R)(\pi_t - \pi^*) + \beta_y (1 - \rho_R)(y_t - y^*). \tag{13}
\]
Appendix A.4  Extension: International Equity Portfolios

We consider an extension of our model that allows for trade in international equities of firms in the traded-goods sector. Let \( s_t(h) \) and \( s_t(f) \) be the fraction of Home firm \( h \)'s and Foreign firm \( f \)'s equities respectively held by the Home household and denote by \( Q_t(h) \) and \( Q_t(f) \) the price of those equities. So, the Home household’s budget constraint is modified as follows

\[
M_t(j) + B_{t+1}(j) + \int Q_t(h)s_t(h,j) + \int \varepsilon_t Q_t(f)s_t(f,j) \leq M_{t-1}(j) + (1 + i_t)B_t(j) + R_tK_t(j) + \\
+ \int (Q_t(h) + d_t(h))s_{t-1}(h,j) + \int \varepsilon_t (Q_t(f) + d_t(f))s_{t-1}(f,j)
\]

where \( d_t(h) \) and \( d_t(f) \) are Home and Foreign firms dividends and where \( \varepsilon_t \) represents the nominal exchange rate.

For the calibration of this version of the model, we keep the values of the discount factor, the labor share in traded (nontraded) goods production, and the capital depreciation rate to those in Table A1. We calibrate the remaining parameter values as follows. We set the weight on traded goods, \( a_T \), to generate a 10 percent import share in steady state. Similarly, the steady State weight on Home traded goods, \( \bar{a} \), so that the consumption of nontraded goods make up 50 percent of total consumption. We assume markups in all sector of 20 percent and that prices change on average every four quarters. We set the traded-non traded goods’ elasticity of substitution to 0.74 and the trade elasticity to 0.3. The parameters of the interest-rate rule are set as follows: \( \rho_R = 0.84, \beta_x = 2.19, \) and \( \beta_y = 0.3 \). Finally, we set the investment adjustment cost to 50, the mid range of our interval in Table A1.

Under this calibration of the model and postulating an 85 percent home bias in equity portfolios, we find that a productivity shock leads to an appreciation of the currency in real terms and a rise in Home consumption relative to consumption abroad, as shown in Figure A2.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor, $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Labor share in traded goods production, $\xi$</td>
<td>0.39</td>
</tr>
<tr>
<td>Labor share in nontraded goods production, $\zeta$</td>
<td>0.44</td>
</tr>
<tr>
<td>Capital depreciation rate, $\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Weight on traded goods, $a_T$</td>
<td>[0.75, 0.95]</td>
</tr>
<tr>
<td>Steady State weight on Home traded goods, $\overline{\pi}$</td>
<td>[0.4, 0.6]</td>
</tr>
<tr>
<td>Nontraded and traded goods steady state markup, $\frac{\theta_N}{\theta_N-1} = \frac{\theta_T}{\theta_T-1}$</td>
<td>[1.1, 1.2]</td>
</tr>
<tr>
<td>Price adjustment costs, $\kappa_N^p, \kappa_T^p, \kappa_T^{*p}$</td>
<td>[0, 187]</td>
</tr>
<tr>
<td>Trade elasticity, $\frac{1}{1 - \rho}$</td>
<td>[0.5, 5]</td>
</tr>
<tr>
<td>Traded-non traded goods’ elasticity of substitution, $\frac{1}{1 - \phi}$</td>
<td>[0.5, 1.5]</td>
</tr>
<tr>
<td>Interest-rate smoothing, $\rho_R$</td>
<td>[0.25, 0.99]</td>
</tr>
<tr>
<td>Coefficient on inflation, $\beta_\pi$</td>
<td>[1.05, 3]</td>
</tr>
<tr>
<td>Coefficient on output, $\beta_y$</td>
<td>[0, 0.7]</td>
</tr>
<tr>
<td>Investment adjustment cost, $b$</td>
<td>[0, 100]</td>
</tr>
</tbody>
</table>
Figure A1: Model's Variable Responses

Productivity Shocks

Demand Shocks

*The figures report the 95 percent confidence intervals over 1 million draws of parameters over the ranges specified in Table A1, following Home and Foreign productivity and demand shocks.*
Figure A2. Responses to a Home Productivity Shock With Equity Portfolios
Appendix B  Data description and sources (Web appendix)

United States
Labor productivity: Index of output per hour of all persons in manufacturing sector, seasonally adjusted, 1992 = 100 (Bank of International Settlements and Dept. of Labor).

Manufacturing output: Index of industrial production in manufacturing, seasonally adjusted, 2000 = 100 (Federal Reserve Board)

Consumption: Private final consumption expenditure, volume in national currency, seasonally adjusted (OECD, Economic Outlook Database).

Nominal GDP: Gross domestic product, value, market prices in national currency, seasonally adjusted (OECD, Economic Outlook Database)

Net exports: Nominal net exports of goods & services, value in national currency, seasonally adjusted (OECD, Economic Outlook Database)

Real imports and exports: Real imports and exports of goods, national currency, seasonally adjusted, 2000 = 100 (NIPA, Table 4.2.3)

PPI index: Producer price index of manufactured products, seasonally adjusted, 2000 = 100 (OECD, Main Economic Indicators Database)

CPI total: Consumer price index all items, seasonally adjusted, 2000 = 100 (OECD, Main Economic Indicators Database)

CPI services: Consumer price index for services less energy services, seasonally adjusted; 1982-84 = 100, monthly converted to quarterly averages (BLS)

Export deflator: Exports of goods, deflator, seasonally adjusted, national accounts basis; 2000 = 100 (OECD, Economic Outlook Database)

Short-term rate: Federal Fund Rate, quarterly (IMF, International Financial Statistics)


US Foreign assets and liabilities: Valuation adjusted US net foreign assets, gross foreign assets and gross foreign liabilities (Gourinchas and Rey (2007), Appendix B)

CPI-based real exchange rate: Index of ratio of US CPI (total) to aggregate CPI (total) of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1970q1 = 100 (authors calculations based on OECD, Economic Outlook Database)

PPI-based real exchange rate: Index of ratio of US PPI (manufacturing) to aggregate PPI (manufacturing) of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1971q1 = 100 (authors calculations based on OECD, Economic Outlook Database)
Terms of trade: Index of ratio of US export deflator (goods and services) to aggregate export deflator (goods and services) of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1970q1 = 100 (authors calculations based on OECD, Economic Outlook Database)

Relative stock prices: Index of ratio of US stock prices to aggregate stock prices of 9 OECD countries, all in current US dollars, weighted with GDP shares at annual PPP values, 1970q1 = 100 (authors calculations based on IMF, International Financial Statistics)

Rest of the world

The rest of the world comprises Japan, Germany, UK, Italy, France, Canada, Australia, Sweden and Ireland. This choice was dictated by data availability regarding hourly productivity in manufacturing.

Individual country’s variables were aggregated, first by taking quarterly growth rates to remove national basis effects; then cross-country average growth rates were computed with weights based on each country’s GDP share in the 9-country aggregate calculated at annual purchasing power parity (PPP) values. Average growth rates were then cumulated starting from the initial base year to obtain levels.

Annual PPP-based GDP shares are from the IMF’s World Economic Outlook Database from 1980; before 1980 they were computed directly on the basis of annual GDP at PPP values from the OECD’s Economic Outlook Database.

Labor productivity: Aggregate of country-specific indexes of output per hour of all persons in manufacturing sector, seasonally adjusted, 1970q1 = 100 (authors calculations based on national statistical sources, BIS and IMF)

Manufacturing output: Aggregate of country-specific indexes of industrial production, manufacturing, seasonally adjusted, 1970q1 = 100 (authors calculations based on national statistical sources, BIS and IMF)

Consumption and investment: Aggregate of country-specific private final consumption expenditure, volumes in national currency, seasonally adjusted, 1970q1 = 100 (authors calculations based on OECD, Economic Outlook Database).

Stock prices: Aggregate of country-specific share prices, 1970q1 = 100 (authors calculations based on IMF, International Financial Statistics and BIS (for UK only)).

Table A2
Acceptance rates of reduced forms draws: Individual shocks
Sample is 1974:1-2004:4

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net exports over GDP</td>
<td>98.90%</td>
<td>98.40%</td>
</tr>
<tr>
<td>Real exports</td>
<td>79.90%</td>
<td>95.90%</td>
</tr>
<tr>
<td>Real imports</td>
<td>96.50%</td>
<td>95.10%</td>
</tr>
<tr>
<td>Relative investment</td>
<td>99.70%</td>
<td>98.70%</td>
</tr>
<tr>
<td>CPI-based RER</td>
<td>99.30%</td>
<td>99.10%</td>
</tr>
<tr>
<td>PPI-based RER</td>
<td>99.20%</td>
<td>99.20%</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>99.10%</td>
<td>98.50%</td>
</tr>
<tr>
<td>Exp. Deflator/PPI</td>
<td>90.70%</td>
<td>95.40%</td>
</tr>
<tr>
<td>Share prices</td>
<td>91.70%</td>
<td>96.20%</td>
</tr>
<tr>
<td>Interest rate differential</td>
<td>98.80%</td>
<td>95.40%</td>
</tr>
<tr>
<td>Foreign assets over GDP</td>
<td>96.80%</td>
<td>98.20%</td>
</tr>
<tr>
<td>Foreign liabilities over GDP</td>
<td>94.80%</td>
<td>96.80%</td>
</tr>
<tr>
<td>Net foreign assets over GDP</td>
<td>99%</td>
<td>97.70%</td>
</tr>
</tbody>
</table>
Appendix C  Methodology (Web appendix)

In the structural VAR literature, identification amounts to providing enough restrictions as to solve uniquely for the following decomposition of the $n \times n$ estimated covariance matrix of the reduced-form VAR residuals $\Sigma$ (up to an orthonormal transformation $Q$ such that $QQ^t = I$):

$$\Sigma = A_0 A_0^t.$$ 

This matrix equation defines a one-to-one mapping from the vector of orthogonal structural shocks $V$ to the reduced form residuals $U$, $U = A_0 V$. Because of the orthogonality assumption, and the symmetry of $\Sigma$, at least $\frac{n(n-1)}{2}$ restrictions on $A_0$ need to be imposed.

The $j$-th column of the identification matrix $A_0$, $A_{0,j}$, is called an impulse vector in $\mathcal{R}^n$, as it maps the innovation to the $j$-th structural shock $v_j$ into the contemporaneous impact responses of all the $n$ variables, $\Psi_{0,j}$. With the structural impulse vector $A_{0,j}$ in hand, the set of all structural impulse responses of the $n$ variables up to the horizon $k$, $\Psi_{1,j}, ..., \Psi_{k,j}$ can then be computed using the estimated coefficient matrix $B(L)$ of the reduced-form VAR, $B_1, B_2, ..., B_p$:

$$\Psi_{s,j} = \sum_{h=0}^{s} B_{s-h} \Psi_{h,j}, \quad s \geq 1, B_{s-h} = 0, s - h \geq p;$$

$$\Psi_{0,j} = A_{0,j}.$$

Proposition 1 in Uhlig (2005) shows that any structural impulse vector $A_{0,j}$ arising from a given identifying matrix $A_0$ can be represented as $Pq$, for an appropriate vector $q$ belonging to the hypersphere of unitary radius $S^n \subset \mathcal{R}^n$, and an arbitrary matrix $P$ such that $PP^t = \Sigma$. For instance, natural candidates for the orthogonal decomposition $P$ are either the eigenvalue-eigenvector or the Cholesky decomposition of $\Sigma$.

Our procedure to obtain estimates of impulse responses consistent with a given set of assumed sign restrictions can be described as follows. Economic theory can be brought to bear, as in Uhlig (2005) or Dedola and Neri (2007), to attribute all the probability mass to the event that the responses of $m \leq n$ variables (e.g., relative labor productivity, relative output, and so on) to the specific structural shock of interest have a given (positive or negative) sign for $s \leq k$ quarters. For instance, Uhlig (2005) appeals to standard monetary theory and assumes that a contractionary monetary policy shock in the US uniquely brings about a hike in the federal funds rate, a drop in the price level and a contraction in money demand (nonborrowed reserves).

Thus, on the basis of theoretical insights, a-priori a non-zero probability is attributed only to structural impulse vectors $A_{0,j}$ which (for a given reduced-form estimate of the VAR) yield impulse responses $\Psi_{s,j}$ whose signs are consistent with the assumed restrictions. It is important to stress that sign restrictions allow us to implement natural theoretical assumptions in the identification procedure (e.g., supply-side shocks should move relative quantities and prices in the opposite direction). However they also lead to a plurality of candidate structural impulse responses. Rather than as a shortcoming, this is a potentially important advantage relative to exact exclusion restrictions, such as long-run restrictions,
that could be sensitive to small perturbations to model specification and parameterization.\footnote{For instance, to adapt long-run restrictions to identify manufacturing shocks in an international context, Corsetti et al (2008c) had to assume that productivity shocks are exactly nonstationary and the only source of a stochastic trend in both the level of labor productivity, and its differential across countries. This amounts to assume permanent differences in international productivity levels. Most importantly, in contrast with the kind of sign-restrictions we use in this paper, long-run restrictions require heavy reliance on unit root tests to buttress key specification assumptions in all the variables included in the VARs.}

As argued by Uhlig (2005), the fact that the Bayesian approach views the VAR parameters as random variables makes it particularly suited to interpreting and implementing sign restrictions. From a Bayesian point of view, on the one hand, the approach amounts to attributing zero probability to reduced-form parameter realizations for which impulse responses contravene the assumed set of sign restrictions. On the other hand, all the impulse responses from the same reduced-form realization that satisfy those restrictions are attributed the same probability a priori. Thus, one can use standard Bayesian methods for estimation and inference, to obtain measures of the uncertainty about estimated impulse responses.

Formally, (i) under a standard diffuse prior on the VAR reduced-form parameters \( B(L) \) and \( \Sigma \), (ii) under a uniform prior on rotation vectors \( q \) in \( S^n \) yielding impulse responses in accord with restrictions, and (iii) assuming a Gaussian likelihood for the data sample at hand, the posterior density of the reduced-form VAR parameters with the type of restrictions we implement will be proportional to a standard Normal-Wishart — whose parameters are known functions of the OLS-MLE estimates of the VAR reduced form (with the proportionality factor being an indicator function equal to one when at least one rotation \( q \) exists yielding impulse responses consistent with the restrictions). This implies that all impulse responses satisfying the restrictions obtained from a given reduced form draw are attributed the same posterior probability, according to the standard Normal-Wishart distribution.

Therefore, it is possible to simulate the posterior distribution of impulse responses consistent with our sign restrictions by jointly drawing from the Normal-Wishart posterior for \( \Sigma, B(L) \) and the uniform for \( q \) over \( S^n \), discarding the impulse responses that violate the restrictions. This could be operationalized by using the following algorithm suggested by Uhlig (2005): for a given estimate of the VAR reduced-form matrices \( \Sigma \) and \( B(L) \), and the associated decomposition \( P \), we draw (a large number of) candidate \( q \) vectors from a uniform distribution over \( S^n \), and compute the associated impulse vector \( A_{0,j} \) and impulse response matrix \( \Psi \), discarding those that do not satisfy the assumed sign restrictions. In practice, the \( q \) vectors are drawn from a multivariate standard normal and normalized with their Euclidean norm to make sure they have unitary length.

To produce two sets of candidate impulse responses which are orthogonal on impact, the required change in the procedure described above is straightforward. We now need to find two vectors, \( q_1 \) and \( q_2 \), both belonging to the hypersphere of unitary radius \( S^n \subset \mathbb{R}^n \), which also satisfy the additional orthogonality condition \( q_1 q_2 = 0 \); then, we can compute the two impulse vectors \( Pq_1 \) and \( Pq_2 \) and the related impulse responses, verifying that they satisfy the sign restrictions for productivity and demand shocks. In practice, the vectors are again drawn from a multivariate standard normal, then orthogonalized and normalized with their Euclidean norm to make sure they have unitary length.