Fiscal Crises, Confidence and Default
A Bare-bones Model
with Lessons for the Euro Area*

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Abstract
Drawing on Calvo (1988), this paper provides a model in which sovereign risk premia are driven by both fundamentals and ‘confidence’ factors, and discuss policy options and trade-offs. The model suggests a rationale for putting a ceiling on interest rates (possibly at a penalty), as a strategy to coordinate markets’ expectations on fundamentals, even when the latter may justify default under some circumstances. Moreover, it suggests that such a policy is likely to strengthen, rather than weaken, the social incentives to implement reforms and budget corrections in crisis countries.

1 Introduction
In many European countries, rising sovereign risk premia have caused a deterioration of debt dynamics, which, if not reversed, may end up calling into question national solvency. One view is that the deterioration of the fiscal outlook priced by markets is entirely due to the worsening of the fundamentals, not matched by a sufficiently strong policy correction. In this perspective, the only way out from fiscal crisis is the implementation of credible reforms that could eventually stem markets’ doubts at their roots. Another view points to the possibility that much of the highly volatile behavior of risk premia be driven by ‘market confidence’, and only weakly related to fundamentals. In this perspective, systemic

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interventions with the explicit goal of correcting mispricing by markets are arguably the most cost-effective response to the crisis, especially in the short run, even in cases in which fundamental budget corrections are necessary.¹

The debate is further complicated by the coexistence of two seemingly irreconcilable notions. The first is the notion that any systemic or bilateral assistance in favor of the crisis countries effectively entails a bailout, and thus cannot but reduce the incentives by domestic policymakers to undertake the necessary reforms — the “moral hazard” problem. The second is the notion that, in absence of systemic interventions reducing the scope for confidence crisis, from the vantage point of the policymakers in the crisis countries the gains from costly reforms are highly uncertain, since risk premia tend to be strongly correlated across borders.

This paper builds upon a well-known contribution by Calvo (1988) on debt default driven by self-fulfilling expectations, to provide a model of fiscal crises reflecting both confidence and fundamentals, and discuss policy options and trade-offs.² We show that the model provides a rationale for putting a ceiling on interest rates as a strategy to coordinate markets’ expectations on fundamentals even when the latter may justify default under some circumstances. To implement a policy ruling out self-fulfilling expectations of default, it is crucial that this ceiling be set at a sufficiently high interest rate, in order to avoid a transfer of resources covering the short-fall of fiscal revenue due to fundamentals — too low an interest rate would effectively amount to a bailout. As is well understood, anticipations of a bailout of this kind would provide incentives to take on more risk ex-ante, giving rise to moral hazard (see e.g. Green 2010 and Prescott 2010 for a recent discussion).

Conversely, we show that an interest rate ceiling appropriately set (i.e. higher than the fundamental rate) is likely to strengthen, rather than weaken, the social incentives to implement reforms and budget corrections in crisis countries. The reason is straightforward. The benefits of reforms are much reduced when policymakers face the prospect of being caught in a self-fulfilling equilibrium, since in this equilibrium the country is forced to default regardless of the soundness of the fundamentals. Without the insurance provided by the interest rate ceiling, the effort to implement reforms cannot be but half-baked — as a function of the perceived probability of ending up in the equilibrium ridden by self-fulfilling prophecies.

Consistent with the goal of discussing fiscal crisis issues in countries participating in a monetary union, the analysis is carried out modelling a small open economy taking the foreign (area-wide) interest rate as given. The model is meant to clarify the basic theoretical foundations of alternative policy strategies in a monetary union, especially as regards the benefits of systemic interventions in the sovereign bond markets, and their effects on the incentive to correct and

reform national fiscal policy.\footnote{While the analysis in this paper is carried out in real terms, in ongoing work we have also worked out a monetary version of the model. All our main conclusions below are confirmed.}

The text is organized as follows. Section 2 builds up the model. Section 3 characterizes multiple equilibria. Section 4 discusses policy options. Section 5 concludes analyzing moral hazard-related issues. Section 6 concludes.

2 The model setup

As in Calvo (1988), the main question in the analysis concerns the determinant of the market price at which a given stock of debt, \( b \), will be bought by private agents at a point in time. In the future, the government who needs to finance the debt service as well as public spending \( G \), is unable to commit to a credible sustainable fiscal plan. Ex post, it may choose to default, partially or fully, on its liabilities. Assuming a small open economy, private agents can invest either in domestic public debt, or an international asset, at the exogenously given "safe" interest rate \( R \). For simplicity, we follow Calvo (1988) in assuming a two-period horizon. However, we explicitly allow for both equilibrium default that is driven by fundamental imbalances, and default that is driven by self-fulfilling expectations.

2.1 Budget constraints and ‘tax capacity’

For the sake of comparison, it is useful to start from Calvo’s original formulation. Given the initial stock of debt, the budget constraint of the government that matters for pricing is that in the second period:

\[
T - G = (1 - \theta) b \tilde{R} + \alpha \theta b \tilde{R}, \quad \alpha, \theta \in (0, 1)
\]

The primary surplus — the difference between taxes \( T \) and government spending \( G \) — finances net debt repayment, the expression on the right hand side. Here, \( \theta \) denotes the extent of the ex-post default of the country. If \( \theta = 0 \), there is no default: the country repays \( b \tilde{R} \), where \( \tilde{R} \) denotes the gross market interest rate on domestic public debt set in the initial period. If a default occurs, repayment is reduced by \( \theta \), but the government also pays a budget cost proportional to the size of the default, \( \alpha \theta b \tilde{R} \).

Raising taxes is costly: taxation results in a dead-weight loss of output indexed by \( z(T) \), which is an increasing and convex function, satisfying standard regularity conditions (see Calvo (1988)). The output cost of taxation, and the default rate, affects the country residents’ budget constraint in the second period, that is

\[
C = [Y - z(T)] - T + KR + (1 - \theta) b \tilde{R}
\]

Consumption is equal to output \( Y \) net of taxes and losses from raising taxes \( z(T) \), plus the revenue from portfolio investment. Consumers own foreign assets \( K \), which yield the (gross) international interest rate \( R \), and public debt, with
net payoffs \((1 - \theta) b \tilde{R}\). For simplicity, the stock of public debt is assumed to be entirely owned at Home, so that consumers’ wealth in the first period must be equal to asset purchases: \(W = K + b\).

A notable property of the Calvo (1988) model is that, under discretion, there is a well-defined upper bound on the country’s willingness to raise taxes which is independent of spending and the interest rate, but varies with the economic costs (or distortions) associated with raising revenue \(z(T)\) and the cost of default \(\alpha\). Under policy discretion, taking \(\tilde{R}\) as given, the ex-post choice of taxes that maximizes private consumption (denoted by \(\hat{T}\)) is:

\[ z'(\hat{T}) = \frac{\alpha}{1 - \alpha} \]

We will refer to \(\hat{T}\) as the ‘tax capacity’ of the government — it should be clear nonetheless that the term ‘capacity’ here is not meant to indicate a technical limit. This ‘tax capacity’ in turns nails down both net output at \(Y - z(\hat{T})\), and the maximum primary surplus that the country can generate in the second period, \(\hat{T} - G\). With this feature of the model in mind, we can now turn to our analysis of default and risk premia.

2.2 Determinants of risk-premia and default

We now take a substantial step away from Calvo’s original formulation. Instead of setting output and spending equal to a constant, we let them vary across two states of the world, \(H\) and \(L\), occurring with probability \(\pi\) and \((1 - \pi)\). The two states of the world are meant to contrast, if only in a stylized way, ‘normal’ and ‘fiscal and macroeconomic stress’ circumstances.

Namely, output varies stochastically between a high (normal) and a low level, dubbed \(Y_H\) and \(Y_L\). Correspondingly, we now write the cost of taxation indexing it to output.

Moreover, output and spending are set such that there is some spare fiscal capacity in the high output case:

\[ \hat{T}_H - G = z' -1 \left( \frac{\alpha}{1 - \alpha} Y_H \right) - G > \tilde{R}b, \]

but default will be a necessary outcome in the low output state, that is:

\[ \hat{T}_L - G < \tilde{R}. \]

\[^4\text{We impose the following two (reasonable and logically consistent) conditions. Namely, for any given } T:\]

\[ z(T, Y_L) > z(T, Y_H), \]

\[ z'(T, Y_L) > z'(T, Y_H). \]

\[^5\text{We proceed under the assumption that (mild) restrictions on the } b, R \text{ and } \alpha, \pi \text{ are satisfied, in order to ensure that the fundamental equilibrium with partial default exists and is unique. As in Calvo (1988), indeed, it may be possible that the fiscal fundamentals are so deteriorated, that no equilibrium trade in bond is possible under rational expectations and unrestricted capital markets.}\]
The driving force of the model is the equilibrium condition in the market for bonds, equating the net rates of return across investments. With risk neutrality, the interest parity condition equalizes expected net returns of international and domestic bonds:

$$\tilde{R}^i \left[ \pi (1 - \theta^i_H) + (1 - \pi) (1 - \theta^i_L) \right] = R$$

where $i$ refers to the equilibrium the private sector coordinates on.

The government budget constraint in period 1 can then be written distinguishing the two states of the world:

$$T^i_H - G - b\tilde{R}^i + \theta^i_H b\tilde{R} (1 - \alpha) = 0 \quad \text{with probability } \pi$$

$$\tilde{T}^i_L - G - b\tilde{R}^i + \theta^i_L b\tilde{R} (1 - \alpha) = 0 \quad \text{with probability } (1 - \pi).$$

Note that, in the model, the government must always exploit all its fiscal capacity $\tilde{T}$ in the low output state, while it may have spare fiscal capacity in the high output state.

3 Interaction between fundamentals and confidence crisis

Following the logic of Calvo’s original contribution, it can be shown that there is an equilibrium in which default only occurs in the low output state $Y_L$, and another equilibrium in which default occurs in both states of the world — with default and interest rates obviously varying across equilibria.

The first equilibrium, that we dub $F$ (for ‘fundamental’), is possible if there is enough spare fiscal capacity

$$\tilde{T}^i_H - G > b\tilde{R}^i = \frac{R}{1 - (1 - \pi) \theta^F_L} > Rb.$$  

In the $F$-equilibrium, no default occurs in the state of the world with a ‘normal’, high level of output, so that $\theta^F_H = 0$. Partial default instead occurs in the low output state:\n
$$T^F_H - G - \frac{R}{1 - (1 - \pi) \theta^F_L} b = 0 \quad \text{with probability } \pi$$

$$\tilde{T}^F_L - G - \frac{1 - (1 - \alpha) \theta^F_L}{1 - (1 - \pi) \theta^F_L} Rb = 0 \quad \text{with probability } (1 - \pi)$$

\footnote{Where we have substituted out for $\tilde{R}^i$ using the equilibrium interest rate:}

$$T^F_H - G - \frac{1 - \theta^H_H (1 - \alpha)}{(1 - \theta^F_L) + \pi (\theta^F_L - \theta^H_H)} bR = 0 \quad \text{with probability } \pi$$

$$\tilde{T}^F_L - G - \frac{1 - \theta^H_L (1 - \alpha)}{(1 - \theta^F_L) + \pi (\theta^F_L - \theta^H_H)} bR = 0 \quad \text{with probability } (1 - \pi)$$
Observe that the second equation above determines $\theta_L^F$, while the first equation determines $T_H < \bar{T}_H$. In this scenario, the default occurs entirely for fundamental reasons, in the sense that investors assign the correct, positive probability to the state of the world in which, facing the drop in output, the domestic government is unable to cut spending below the reduced taxing capacity of the country. The larger this probability, the higher the risk premia the government will pay on its debt: $\left[1 - (1 - \pi)\theta_H^F\right] \bar{R}^F = R$.

In the second equilibrium, dubbed $N$ for ‘non-fundamental’, the risk premia reflect both fundamental and non-fundamental factors. This equilibrium is characterized by

$$
\hat{T}_H - G - \frac{1 - \theta_H^N (1 - \alpha)}{(1 - \theta_L^N) + \pi (\theta_L^N - \theta_H^N)} bR = 0 \quad \text{with probability } \pi
$$

$$
\hat{T}_L - G - \frac{1 - \theta_L^N (1 - \alpha)}{(1 - \theta_L^N) + \pi (\theta_L^N - \theta_H^N)} bR = 0 \quad \text{with probability } (1 - \pi)
$$

Here we have two equations in two unknown rates of default — taxes are always set to capacity. As in Calvo (1988), investors’ expectations force default not only in the low-output state, but also (quite inefficiently) in the high-output state.

4 Policy options to stem confidence crises

To discuss possible policy options, the key observation is that differences in welfare across equilibria are driven by differences in the output losses caused by taxation and default. Namely, in the $N$—equilibrium, the economy is forced to incur deadweight losses that are avoided in the $F$—equilibrium. Specifically, in

$$
\theta_L^F = \frac{\hat{T}_L - G - Rb}{(1 - \pi) \left( \hat{T}_L - G - Rb \right) - (\pi - \alpha) Rb}.
$$

The above equation obviously restricts parameters in order to have $0 < \theta_L^F < 1$. For instance, it is easy to show that we can have an equilibrium with complete default in the low state if the following condition is satisfied:

$$
\frac{\hat{T}_L - G}{\frac{R}{\pi} b} < \bar{R}
$$

$$
\frac{\bar{R}}{\pi} = \frac{R}{\pi}
$$

There are other two possible scenarios. First, the equilibrium may be unique, but this occurs for the special case in which $T_H^F$ happens to coincide with $\bar{T}_H$. Second, as already mentioned, the equilibrium may not exist. If the politically-determined tax (or primary surplus) capacity is low enough — in the model, this is possible when the cost of default is low — there may be no market rate at which the bond market clears. In other words, there is no market risk premia that can be justified by the size of the ex-post primary surplus in period 1, for $\theta \leq 1$. In this scenario, no debt can be sold in the initial period.
the $N$-equilibrium, the increase in the interest rate due to the ‘self-fulfilling attack’ on public debt in the initial period causes the government to raise taxes to capacity also in the high output (normal) state. In the high output state, therefore, consumption and output fall considerably, relative to the ‘no-crisis circumstances’ of the $F$-equilibrium, in which taxation is kept well below capacity. Observe that, under the model’s assumption, what happens in the low-output state does not impinge on welfare differences across equilibria, since default always occurs in that state, and the government is always forced to raise taxation to capacity.

As emphasized by Calvo in his original paper, there is a straightforward policy that can improve welfare. This consists of setting a ceiling $\bar{R}$ on the interest rate on government debt. Such a ceiling should be at least as high as the interest rate in the fundamental equilibrium, $R_F^*$, but sufficiently low as to rule out the bad equilibrium driven in part by self-fulfilling crises:\(^9\)

$$\bar{R}^N > \bar{R} \geq R_F^* = \frac{R}{1 - (1 - \pi) \theta_L^F}.$$ 

Essentially, the goal of this policy is to act as a coordination device shifting market expectations on the fundamental equilibrium.

In principle, the ceiling on rates could be imposed via restrictions on the free circulation of capital and portfolio allocations of households, firms or financial intermediaries. Alternatively, and perhaps in a more timely and effective way, an institution with sufficiently deep pockets can announce and carry out intervention in the market for the country’s debt.

Three observations are worth stressing, each defining an interesting direction for model extensions and future research. First, for this strategy to work, the ceiling should be fully credible. The institution that carries it out must commit to it, and implement it without reacting to changes in market expectations. This is a key precondition, which in practice may not be granted. Recall that the model is derived under the maintained assumption that the fiscal authority of the country lacks credibility in the first place. Doubts about the ability by an independent central bank or an external institution to carry through a cap policy would simply undermine its effects on selecting the virtuous equilibria.

In this respect, it is also important to stress that the credibility of the central bank ceiling is key not to generate adverse expectations — that liquidity support will ultimately result in an ex-post monetary financing through the inflation tax. Not only an intervention lacking credibility would fail to dispel confidence crises: it may even fuel them via self-fulfilling inflationary expectations (see Calvo (1988)).

Finally, stemming confidence crises should not be confused with transferring resources in favor of the national residents to cover a short-fall of fiscal revenue due to fundamentals — the strict definition of a bailout. This would follow if

\(^9\)In the model, it is easy to verify that the ceiling cannot exceed the market rate at which the best response of the government is a strictly positive default rate in the high output state.
the cap rate were to be set below the rate in the $F$--equilibrium:

$$\tilde{R}_N > \tilde{R} \geq \tilde{R}_F > \tilde{R}_{\text{bail-out}}$$

Of course, anticipations of a bailout of this kind would provide incentives to take on more risk ex-ante, giving rise to moral hazard (see e.g. Green 2010 and Prescott 2010 for a recent discussion). In the appendix we provide a simple example in which expectations of such a bailout rate would indeed result in issuing too much debt to begin with.

Conversely, there are strong reasons to expect that setting the interest rate cap strictly above the fundamental rate can actually increase incentives not to postpone budget corrections to the future, and restore confidence. To this issue we turn next.

5 Is stemming confidence crises bound to reduce incentives to reforms?

What are the effects of interventions targeted to stem confidence crises on the incentive for government in a fiscal crisis to undertake actions that would make a default less likely — the “moral hazard” problem? Abstracting from the complexity of this problem (discussed by a vast and rich literature), an often-heard view simply states that any kind of interventions (including those discussed in the previous section), cannot but make things worse, because they undermine the market discipline on the crisis country, weakening the pressure to straighten its fiscal conduct. A simple extension of the model turns out to be effective in unveiling the pitfalls in such a view — a view that runs the risk of blurring key trade-offs relevant to policy decisions.

Let the government choose, in period 0, whether to implement reforms that, while still falling short of restoring perfect credibility, would increase the probability $\pi$ of being in a high-output equilibrium. These reforms could be structural, political or institutional. What matters is that they improve the prospects for the country to end up in the ‘normal’ $Y_H$ state. However, to implement them, policymakers incur a cost which, without loss of generality, can be modelled as a utility cost — reflecting e.g. political considerations, as reforms may create tensions and break former alliances and run against the interests of specific lobbies.

Will reforms be discouraged by a policy of interest rate ceilings, aimed at coordinating expectations on a ‘fundamental’ equilibrium? The answer is no. Actually, the opposite is true. Without specifying a particular functional form for the costs and benefits of reforms, it is straightforward to envision that, in equilibrium, reforms will be undertaken if, at the margin, the expected economic benefits in term of higher consumption are at least as large as their initial cost. In the model, there are positive benefits to reform to the extent that consumption (private and public) is higher in the high output state, even conditional on a confidence crisis. However, reforms that raise the probability of a high output
state produce much larger benefits on consumption and welfare in the virtuous $F$-equilibrium, since only in this equilibrium taxation can be kept below capacity, and tax-distortions contained.

It follows that the benefits of reforms are much reduced when policymakers face the prospect of being caught in the $N$-equilibrium, since in this equilibrium the country is forced to default in both the high and the low output state. Without a shelter from confidence crises, the government always faces the possibility of ending up with taxation at capacity, causing large output losses independently of the realization of the state.

Without insurance, the effort to implement reforms cannot be but half-baked — as a function of the perceived probability of ending up in the equilibrium ridden by self-fulfilling prophecies. Insurance and efforts are strategic complements, as discussed also by Morris and Shin (2006) and Corsetti Guimaraes and Roubini (2005).

6 Conclusion

Fiscal crises can be expected to reflect both fundamentals and confidence factors, whose interactions drive fluctuations in sovereign risk premia. Understanding these interactions is crucial to define the relevant policy trade-offs faced by national and international policymakers. In this paper we have presented a stylized framework to discuss policy options for containing the costs of fiscal crisis. Specifically, we have shown that whether or not a policy of insurance against self-fulfilling attacks exists matters substantially. Unless reforms can eliminate quickly and completely market doubts on public solvency, there are theoretical reasons to believe that efforts to reforms are discouraged by the perspective of a price-debt spiral due to self-fulfilling prophecies.

This is not to deny moral hazard as a key concern. At the root of a fiscal crisis may well lie anticipations of a bailout, prompting governments to ‘save’ on budget and economic adjustment over time, and thus resulting in the accumulation of explicit and implicit liabilities, and economic and financial fragility. But once in the middle of a crisis, denying liquidity assistance is tantamount to exacerbate the costs of the punishment for past behavior — perhaps imparting a good lesson for the distant future, but also jeopardizing the prospects for a successful implementation of domestic reforms in the current circumstances.\footnote{This is also not to deny the possibility that, in some cases, policymakers may not be up to the job. In these cases, replacing incumbent governments would be a necessary step for carrying out successful stabilization.}

The message of this paper is limited to emphasize a specific but important point. In a situation of crisis and contagion, with governments lacking credibility, instruments to counteract confidence crises are likely to be a necessary component of a comprehensive policy strategy of fiscal and economic stabilization, at systemic as well as at country level. For such a strategy to work, of course, it should be fully credible. Doubts about the ability by an independent central bank or an external institution to carry through a cap policy would un-
dermine its effects on selecting the virtuous equilibria, and even fuel adverse expectations — that liquidity support will ultimately result in an ex-post monetary financing through the inflation tax. Needless to say, these possibilities define an intriguing area of academic and policy research.

References


7 Appendix

To derive tax capacity, write the problem:

\[ \text{Max } C = [Y - z(T, Y)] - T + KR + b\bar{R} - \theta b\bar{R} \]

By the budget constraint of the government

\[ T - G = b\bar{R} - \theta b\bar{R} + \alpha \theta \bar{R} \]

\[ \theta b\bar{R} = \frac{G - T + b\bar{R}}{1 - \alpha} \]

we can rewrite the above expression as

\[ \text{Max } C = [Y - z(T, Y)] - T + KR + b\bar{R} - \frac{G - T + b\bar{R}}{1 - \alpha} \]

The first order condition of the problem is:

\[ z'(T, Y) = \frac{\alpha}{1 - \alpha} \]

To derive \( \theta_L^E \) in the \( F \)-equilibrium, just rewrite the equilibrium expression in the Low output state

\[ \hat{T}_L - G - \frac{bR \left[ 1 - \theta_L^E \left( 1 - \alpha \right) \right]}{\left( 1 - \theta_L^E \right) + \pi \theta_L^E} = 0 \]

\[ \left[ (1 - \theta_L^E) + \pi \theta_L^E \right] \left[ \hat{T}_L - G \right] - bR \left[ 1 - \theta_L^E \left( 1 - \alpha \right) \right] = 0 \]

\[ \left[ \hat{T}_L - G \right] - \theta_L^E \left[ (1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) \right] - bR = 0 \]

Rearranging the above yields.

\[ \theta_L^E = \frac{\hat{T}_L - G - bR}{(1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha)} \]
The level of taxation in the High output state easily follows:

\[
T^F_H = G + \frac{bR}{1 - \theta^F_L + \pi \theta^F_L} = \\
= G + \frac{bR}{1 - (1 - \pi) \frac{\hat{T}_L - G - bR}{(1 - \pi) (T_L - G) + bR (1 - \alpha)}} = \\
= G + \frac{bR \left[ (1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) \right]}{(1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) - (1 - \pi) \left( \hat{T}_L - G - bR \right)} = \\
= G + \frac{bR \left[ (1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) \right]}{(1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) - (1 - \pi) \left[ bR \right]} = \\
= G + \frac{\left[ (1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) \right]}{(1 - \alpha) + (1 - \pi)} \\
\]

This tax rate is decreasing in the probability \( \pi \)

\[
\frac{\partial T^F_H}{\partial \pi} = \left[ - \left( \hat{T}_L - G \right) \right] \left[ (1 - \alpha) + (1 - \pi) \right] + \left[ (1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) \right] = \\
= \left[ \left( \hat{T}_L - G \right) \right] \left[ (1 - \alpha) + (1 - \pi) \right] + \left[ (1 - \pi) \left( \hat{T}_L - G \right) + bR (1 - \alpha) \right] = \\
= \left[ \left( \hat{T}_L - G \right) \right] \left[ (1 - \alpha) \right] + bR (1 - \alpha) = \\
= (1 - \alpha) \left[ bR - \left( \hat{T}_L - G \right) \right] \leq 0 \\
\]

The weak inequality follows from the restrictions on parameters which by construction posit some spare taxing capacity in equilibrium.
Here we formalize the idea that reform and insurance are strategic comple-
ments. First, we consider the case of a reform that raises the probability of
being in the high output equilibrium. A second example, we endogenize the
initial choice of debt $b$.

Define a fixed costs of reform $\Gamma$. Let $q$ denote the exogenous probability
of the $F-$equilibrium. The ex-ante value of reform can be assessed by taking the
difference in the expected welfare with and without the reform

$$
q \cdot [\pi (\Gamma) \, V \left( C_H (\Gamma) \right) - \pi V \left( C_H \right)] - q \cdot \pi V \left( C_H \right) + \pi (\Gamma) \, V \left( \tilde{C}_H \right) + (1 - \pi (\Gamma)) \, V \left( \tilde{C}_L \right) =
$$

$$
q \cdot [\pi (\Gamma) \, V \left( C_H (\Gamma) \right) - \pi V \left( C_H \right)] + q \cdot (1 - \pi (\Gamma) - 1 + \pi) \, V \left( \tilde{C}_L \right) +
$$

$$
(1 - q) \cdot \left( (\pi (\Gamma) - \pi) \, V \left( \tilde{C}_L \right) \right) +
$$

$$
(1 - q) \cdot \left( (1 - \pi (\Gamma) - 1 + \pi) \, V \left( \tilde{C}_L \right) \right) =
$$

$$
q \cdot \left( (\pi (\Gamma) - \pi) \, V \left( \tilde{C}_L \right) \right) + (1 - \pi (\Gamma)) \, V \left( \tilde{C}_L \right) =
$$

So, we need to study

$$
q \cdot \left( (\pi (\Gamma) - \pi) \, V \left( \tilde{C}_L \right) \right) + (1 - \pi (\Gamma)) \, V \left( \tilde{C}_L \right) =
$$

$$
q \cdot \left( (\pi (\Gamma) - \pi) \, V \left( \tilde{C}_L \right) \right) + (1 - \pi (\Gamma)) \, V \left( \tilde{C}_L \right) =
$$

which is positive, since $\pi (\Gamma) > \pi$ and $V \left( C_H (\Gamma) \right) > V \left( C_H \right)$ due to the fact
that in the $F-$equilibrium the risk premium is lower.

Now, what happens to the value of this expression if insurance turns $q < 1$
into $q = 1$. Taking the difference, reform is more attractive conditional on
insurance if the following is positive:

\[
\begin{align*}
\left[\pi\left(\Gamma\right) \left[ V\left(C^f_H\left(\Gamma\right)\right) - V\left(\tilde{C}^f_L\right)\right] - \pi \left[ V\left(C^f_H\right) - V\left(\tilde{C}^f_L\right)\right]\right] \\
- \left[\left(\pi\left(\Gamma\right) - \pi\right) \left[ V\left(\tilde{C}^N_H\right) - V\left(\tilde{C}^N_L\right)\right]\right] = \\
= \left[\pi\left(\Gamma\right) V\left(C^f_H\left(\Gamma\right)\right) - \pi V\left(C^f_H\right)\right] - \left(\pi\left(\Gamma\right) - \pi\right) V\left(\tilde{C}^f_L\right) \\
- \left(\pi\left(\Gamma\right) - \pi\right) \left[ V\left(\tilde{C}^N_H\right) - V\left(\tilde{C}^N_L\right)\right] = \\
\left[\pi\left(\Gamma\right) V\left(C^f_H\left(\Gamma\right)\right) - \pi V\left(C^f_H\right) + \pi\left(\Gamma\right) V\left(C^f_H\right) - \pi V\left(C^f_H\right)\right] \\
- \left(\pi\left(\Gamma\right) - \pi\right) V\left(\tilde{C}^f_L\right) - \left(\pi\left(\Gamma\right) - \pi\right) \left[ V\left(\tilde{C}^N_H\right) - V\left(\tilde{C}^N_L\right)\right] = \\
\pi\left(\Gamma\right) \left[ V\left(C^f_H\left(\Gamma\right)\right) - V\left(C^f_H\right)\right] + \left(\pi\left(\Gamma\right) - \pi\right) \left[ V\left(C^f_H\right) - V\left(\tilde{C}^f_L\right)\right] \\
- \left(\pi\left(\Gamma\right) - \pi\right) \left[ V\left(\tilde{C}^N_H\right) - V\left(\tilde{C}^N_L\right)\right] = \\
\pi\left(\Gamma\right) \left[ V\left(C^f_H\left(\Gamma\right)\right) - V\left(C^f_H\right)\right] + \left(\pi\left(\Gamma\right) - \pi\right) \left[ V\left(C^f_H\right) - V\left(\tilde{C}^N_H\right)\right] > 0.
\end{align*}
\]
Here we look at a multiperiod version of the model, dropping the assumption of stochastic output in the last period. Assume the government budget constraint in period 1 and 2 (the latter denoted with a prime) is:

\[ b + T = G \]  
\[ T' = (1 - \theta) \bar{R}b + G' + \alpha \theta \bar{R}b, \]

where without loss of generality the government in the first period only decides taxes and debt issuance. Under linear utility, let’s start assuming that consumers only care about consumption in period 2 and face the following budget constraints:

\[ W = W_0 R + z (\tau) - \tau \]  
\[ C = Y + WR + \left( (1 - \theta) \bar{R} - R \right) b - z (\tau') - \tau'; \]

bond interest rates are determined according to interest parity:

\[ (1 - \theta) \bar{R} = R. \]

The solution to the government problem in period 2 is the same as before, namely:

1. a unique equilibrium with \( \theta = 0 \) and
\[ T' = \hat{T} = z'^{-1} \left( \frac{\alpha}{1 - \alpha} \right) = G' + Rb \]

2. two equilibria if \( \hat{T} > (G' + Rb) \)
\[ \theta = 0, T' = G' + Rb \]
\[ \theta > 0, T' = \hat{T} = Rb + G' + \alpha \frac{\theta}{1 - \theta} Rb \]

3. No equilibrium if positive debt
\[ \hat{T} < G' + Rb, \]

implying that the maximum amount of debt that can be sold in period 1 is limited by the ceiling on the present discounted value of the primary surplus in period 2.

\[ R^{-1} \left( \hat{T} - G' \right) = b. \]

The problem for the benevolent government in period 1 is to maximize (expected) consumption in period 2 by choosing \( b \) and \( \tau \), taking into account the behavior of the future government and subject to its budget constraint:

\[ \max_b E \frac{C}{1 + \delta} = \max_b (1 + \delta)^{-1} \left\{ -\left[ z (G - b) + (G - b) R + \left[ -q [z (G' + Rb') + G' + Rb'] - (1 - q) \left[ z \left( \hat{T} \right) + \hat{T} \right] \right] \right\} \]
\[ b \leq R^{-1} \left( \hat{T} - G' \right), \]
where in the last period with with probability $q$ and $1-q$:

$$
\begin{align*}
\theta &= 0, T' = G' + Rb \\
T' &= \tilde{T} = Rb + G' + \alpha \frac{\theta}{1-\theta} Rb.
\end{align*}
$$

This assumes that there is an exogenous variable (sunspot) that for any given quantity of debt sold $b \leq R^{-1} \left( \tilde{T} - G' \right)$, coordinates with probability $q$ investor expectations in believing that there will be no default and $\tilde{R} = R$, or with probability $1-q$ that there will be default and $\tilde{R} > R$. The government decides how much debt to sell before this sunspot variable is observed. As noted in the original paper by Calvo:

“the way the above “game” has been structured is consistent with situations in which the government at time 1 auctions off the quantity of debt $b$, and lets the market free to determine the associated interest factor.”

The FOC of this problem is:

$$
\begin{align*}
z'(G - b) + 1 & \geq q [z'(G' + Rb) + 1] \\
\quad b & \leq R^{-1} \left( \tilde{T} - G' \right).
\end{align*}
$$

It is easy to see that the higher the probability of the equilibrium without default $q$, the lower the debt $b$ issued by the government and the higher the welfare. Specifically, notice that under commitment $q = 1$ and the optimal solution would be to equate tax distortions across the two periods:

$$
\begin{align*}
z'(T) &= z'(T') \\
T &= G - b \\
T' &= G' + Rb \\
b' &\leq R^{-1} (\tau^* - g').
\end{align*}
$$

This establishes that a mechanism ensuring that the good equilibrium prevails in the second period would be optimal even ex-ante. Conversely, setting a rate below $R$ would imply that the government opportunistically issues too much debt in the first period relative to the first best.