Sovereign Risk

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The questions

In analyzing gains from trade in financial assets in previous notes, we implicitly assumed that financial contracts are always honoured. Without a supranational authority with legal power, however, countries/individuals may not be able to resolve international disputes on financial contracts.

- **Enforcing financial contracts across borders** is more difficult than within a country. More than the ‘ability to pay’, what matters is the ‘willingness to pay’ of the debtor.

- **The conditions (states of the world) on which contracts could be written typically are not verifiable.** E.g., the way contracts are written may create incentives to misrepresent the true state/conditions of the debtor. Debtors may undertake actions that affect ex-post the payments to creditors (debtors’ moral hazard).

What are consequences for the development and efficiency of international financial markets? In these notes we will address this question first with a numerical model, than with a formal model. We conclude with an appendix on debtors’ moral hazard.
1 An introductory model/example
   - Consumption smoothing with contingent debt
   - Sovereign risk and debt sustainability

2 A textbook model
   - An economy with a continuum of states
   - The perfect-risk sharing benchmark
   - Asset trade and costly default
   - Reputation costs

3 Sovereign risk and international financial markets
   - Implications for international financial contracts
   - Consequences for int’l asset trade and risk sharing

4 Appendix: Consumption smoothing with non-contingent debt
An introductory model/example

To introduce sovereign risk, we start by reconsidering international borrowing and lending through contingent debt contracts, that allow residents in a country to smooth consumption not only over time (as in the current account model), but also across circumstances—thus they can insure income and consumption against adverse realization of domestic shocks.

We will use stylized model/example taken from Broner, Martin and Ventura (2012) to introduce the main issues.

Consider a small-open economy, with population normalized to 1. The economy lasts two periods (0,1). It is fully integrated in international capital markets. For simplicity, assume:

1. $R = 1$, the international real interest rate is equal to zero.
2. International investors are risk neutral and do not discount the future.
   a. the price of any asset is equal to the expected cash flow from it.
An introductory model/example

Instead of assuming a representative agent, residents have heterogeneous income risks.

- In period 0, all residents $i$'s have the same income $y_{i,0} = .5$.

- In period 1, a resident $i$ can be 'lucky' and get $y_{i,1} = 2$, or 'unlucky' and get $y_{i,1} = 1$, with probability .5.

\[
y_{i,0} = .5 \quad \text{and} \quad y_{i,1} = \begin{cases} 
2 & \text{with probability } 1/2 \\
1 & \text{with probability } 1/2
\end{cases}
\]

Note that, first, all residents are symmetrically poorer in period 0 than in period 1: hence they have an incentive to ‘borrow’ (homework: assuming $U = \ln(C)$ show that $R^{FA} > R$). Second, while period 1 income is random from an individual perspective, there is no aggregate uncertainty: half the population has high income, the other half has low income.
**Contingent debt:** Consider the possibility for a resident $i$ to issue debt $\tilde{b}_0$ promising a differentiated cash flow in period 1, contingent on the ex-post realization of her individual income. Namely, in period 1, the debt contract pays $x^H_i$ if the individual turns out to have high income, and $x^L_i$ otherwise.

The individual preferences, endowment and budget constraint are now:

\[
U(c_{i,0}) + EU(c_{i,1})
\]

\[
c_0 = y_{i,0} + \tilde{b}_0 \quad \text{and} \quad c_{i,1} = \begin{cases} 
    c^H_{i,1} = y^L_{i,1} - x^H_i \\
    c^L_{i,1} = y^L_{i,1} - x^L_i
\end{cases}
\]

Since international investors and risk neutral and do not discount the future, $\tilde{b}_0$ will be equal to the expected payoff:

\[
\tilde{b}_0 = \frac{1}{2} x^H_i + \frac{1}{2} x^L_i.
\]
The optimal borrowing problem can be written as

$$\max_{x_1^L, x_1^H} U \left( y_{i,0} + \left( \frac{1}{2} x_i^H + \frac{1}{2} x_i^L \right) \right) + \left[ \frac{1}{2} U (y_{i,1}^H - x_1^H) + \frac{1}{2} U (y_{i,1}^L - x_1^L) \right]$$

has two first order condition, one for each state

$$U' \left( .5 + \tilde{b}_0 \right) \frac{\partial \tilde{b}_0}{\partial x_1^H} = \frac{1}{2} U' \left( 2 - x_1^H \right)$$

$$U' \left( .5 + \tilde{b}_0 \right) \frac{\partial \tilde{b}_0}{\partial x_1^L} = \frac{1}{2} U' \left( 1 - x_1^L \right)$$

where $\frac{\partial \tilde{b}_0}{\partial x_1^H} = \frac{\partial \tilde{b}_0}{\partial x_1^L} = \frac{1}{2}$. 
Consumption smoothing with contingent debt

It is easy to see that, if residents pledge a contingent cash flow equal to $x_i^H = 1 > 0$ if lucky, and $x_i^L = 0$ otherwise, they can **smooth consumption completely across time and circumstances**. In period 0, they borrow .5 from international risk neutral investors:

$$\tilde{b}_0 = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = .5.$$ 

raising current consumption from .5 to 1. In period 1, the debt contract fully insures their consumption from output risk.

$$c_0 = y_{i,0} + \tilde{b}_0 = .5 + .5 = 1 \quad \text{and} \quad c_{i,1} = \begin{cases} c_{i,1}^H = y_{i,1}^L - x_i^H = 2 - 1 = 1 \\
c_{i,1}^L = y_{i,1}^L = 1 \end{cases}$$

- Given the properties of the utility function, smoother consumption will increase individual and thus global welfare, relative to the case of non-contingent debt.
Sovereign risk

Will the debt contracts above, either with or without risk sharing, be sustainable?

- Within a country, creditors can rely on judicial and law enforcement institutions and force debtors to honour, at least in part, their liabilities.

- There is no international authority with the power to enforce contracts among households, firms and public institutions residing in different sovereign states.

What are the consequences for cross-border asset trade? Without loss of generality, we will focus our discussion on the case of contingent debt $\tilde{b}_0$. 
Sovereign risk and debt sustainability

With no international institution enforcing contracts across borders, ex-post ‘lucky’ residents have no incentive to honour their contracts and make the payment $x_i^H$ to their foreign creditors in period 1. These may try to “sue delinquent debtors in the debtors’ home country,” but a welfare-maximizing sovereign government will have no incentive to rule against domestic residents and/or force them to pay: the national social welfare will be higher if sovereign debt is repudiated.

Here is the key implication: anticipating this outcome, foreign investors know that (whatever the ex-ante promise $x_i^H$) ex post they will get $x_i^H = 0$. So, under rational expectations, they will not be willing to lend/enter the contract since:

$$\tilde{b}_0 \leq \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0$$

The residents in the country will simply be unable to borrow and share output risk through international financial markets: domestic consumption will be at “financial autarky” levels.
Sovereign risk and debt sustainability

Why can countries lend and borrow internationally then? The answer must be that default entails some cost. Let $K \geq 0$ denote these costs in terms of period 1 output losses.

- If default is costly $K > 0$, rational domestic residents will prefer “to pay” rather than “defaulting” as long as debt obligations are below $K$.
  - In our example, if $K = .5$, the promise to pay $x_i^H$ will be credible up to $K = .5$. International investors will be willing to lend up to:
    \[
    \tilde{b}_0 \leq \frac{1}{2} \cdot K + \frac{1}{2} \cdot 0 = \frac{1}{2} \cdot .5 = .25
    \]

- With positive default cost, hence, there will be some scope for risk sharing.
  - in our example, if $K = .5$, consumption will be
    \[
    c_0 = .75 < c_{i,1}^H = 1.5 < c_{i,1}^L = 1,
    \]
    smoother than under financial autarky.

- If default costs are sufficiently high, consumption will actually be perfectly smoothed
  - in our example, risk sharing is complete if $K \geq 1$. 
What are the ‘costs’ of default?

There are at least two leading views dominating this highly controversial issue.

1. Exclusion from international capital markets.
   - After defaults, investors may (and actually do) refuse to lend to the country for a prolonged period of time.
   - Long and difficult legal cases abroad disrupt residents’ trade and business in international markets.

2. Disruption of domestic markets:
   - Since it is difficult to separate ‘external’ and ‘domestic’ debt in practice, a default also hits domestic residents (households, firms and banks).
   - In anticipation and per effect of default, the country experiences a sharp rise in macroeconomic uncertainty. Investors anticipate that the government will need to raise taxes (which taxes will be raised?), or cut spending (which type will be cut?). It may need to intervene to save some bank/industry (which one will be saved?). Uncertainty raises corporate risk, reducing access to credit; makes banks ‘reluctant to lend’; discourages spending and investment, as firms and households may prefer to ‘wait’ until the crisis is resolved before starting new projects or spending plans.
A model from Obstfeld and Rogoff
OR, Chapter 6

The world economy consists of

1. a debtor (Home) country, populated by a continuum of identical agents treated as a single entity, i.e. without distinguishing the government from the private sector;

2. international investors who operate in competitive international financial markets, are risk neutral and always honor their liabilities, even if domestic investors may not.

Think of international investors as British residents, and contracts written under British law—foreign residents can take British lenders to British courts.

Focus the analysis on one period only. Let $s$ denote the state of nature, each occurring with probability $\pi(s)$ during the period. The endowment of Home residents is:

$$Y(s) = Y + \epsilon(s)$$

At the beginning of the period, before uncertainty is realized, Home residents can buy insurance from international markets, i.e. they can enter contracts specifying contingent cash flows $d(s)$ that they pay (if positive) or receive (if negative) conditional on the state of the economy.
The residents’ consumption (=**budget constraint**) will therefore be

\[ C(s) = Y + \epsilon(s) - d(s) \]

Risk neutral investors will lend to the country as long as they anticipate non-negative profits:

\[ \sum_s \pi(s) d(s) \geq 0 \]

This is a ‘**participation constraint:**’ if not satisfied, international markets will not accept the trade. With competitive international market, the expected profits will actually be driven to zero

\[ \sum_s \pi(s) d(s) = 0. \]

In our example, the cash flows \( d(s) \) can be made contingent on all states of nature. If an international authority could enforce cross-border contractual payments, the country could ensure its output risk completely.
The perfect-risk sharing benchmark

The following characterizes the optimal contracts under the assumption that these are enforceable.

The optimal contracts consist of scheduled payments $d(s)$ that solve $\text{Max } U(C)$ subject to (a) the budget constraint, and (b) the participation constraint by international investors (without loss of generality, imposed with an equality sign). The Lagrangian is

$$\text{Max}_{d(s)} \sum_{s} \pi(s) U(Y + \epsilon(s) - d(s)) + \lambda \sum_{s} \pi(s) d(s)$$

(2)

where $\lambda$ is the constant Lagrangian multiplier associated to the participation constraint. The first order condition

$$U_C = \lambda$$

implies constant consumption across states of nature

$$C(s) = Y$$

With risk neutral international investors, home residents can diversify country-idiosyncratic output risk completely (note that state prices are ‘actuarially fair’).
Contractual payments coincide with output shocks (can be negative or positive)

Will the contract be honoured? In the simple economy above, ex post there is a clear incentive for domestic residents not to honour their contracts when shocks to output are positive, implying that they need to pay resources to international investors.
If contracts are not enforceable, in our one-period economy, residents have no incentive/reason to pay when output shocks are positive. Rational investors anticipate this. As a result, \textit{markets become endogenously incomplete.} To see this, add to the problem (2) an additional constraint, the \textbf{incentive-compatibility constraint}, taking the form:

\[ d (s) \leq 0. \]

Combining this with the \textbf{participation constraint}, without going to a formal derivation, it is apparent that the only possible solution is, trivially,

\[ d (s) = 0 \]

and no financial trade takes place. Even if, in principle, international financial contracts could provide full output insurance to Home residents, the incentive for a debtor to default on payments ex post prevents trade from taking place. De facto, the country is in financial autarky, and its risk-averse residents are worse off.
Asset trade with costly default

Some trade may nonetheless take place if default on payments cause residents to suffer some (resource or welfare) costs.

As discussed above:

- the lenders have some way to punish the delinquent country, in the current or future periods;
- default may interfere with and impair the normal functioning of domestic markets, with disruptive effects on production and welfare, or
Suppose foreign lenders can seize assets or goods belonging to a country defaulting on its payments. Let $K$ denote the value of the sanction. Clearly, it is not rational for the country to default if contract payments are lower than $K$. The ‘incentive compatibility constraint’ (ICc) is

$$d(s) \leq K.$$ 

To study the optimal contract in the presence of sanctions, write:

$$\max_{d(s)} \sum_s \pi(s) \left\{ U(Y + \epsilon(s) - d(s)) \right\} + \lambda d(s) + \gamma(s)(K - d(s))$$

where $\lambda$ is as defined above, and $\gamma(s) \geq 0$ are the Lagrangian multiplier associated with the ICc in each state $s$ (there are $S$ multipliers).
Asset trade with costly default

Applying the Kuhn-Tucker theorem, the first-order condition w.r.t. to $d(s)$ is:

$$U_C(C(s)) = \lambda - \gamma(s).$$

The ‘complementary slackness’ condition (indexing whether the ICc is binding in $s$) is

$$\gamma(s)(K - d(s)) = 0.$$

- When $d(s) < K$, the ICc is not binding, and the country honours its liabilities: $\gamma(s) = 0$ and the marginal utility and hence consumption is constant ($U_C = \lambda$).
- However, different from the case of full insurance, $C$ cannot be equal to average output $Y$. To see why, use the budget constraint

$$d(s) = (Y + \epsilon(s) - C)$$

If $C = Y$, the country should accept to make payments $d(s)$ equal to $\epsilon(s)$ for any realization of the shock, in violation of the ICc.
Asset trade with costly default

Because of sovereign risk, the cash flows \( d(s) \) will differ from the stochastic realization of output by some constant. Let define this constant \( d \).

- For large enough output realizations, the ICc will be binding, so that \( \gamma(s) \neq 0 \), residents will default and suffer sanction costs. Hence their consumption will be lower than exogenous output by \( d(s) = K \).

Denote with \( \epsilon(\tilde{s}) \) the smallest level of output \( Y + \epsilon(\tilde{s}) \) at which the country rationally defaults. Allowing for the sanctions, the “cash outflows” from (or the cash lost by) the debtor country is

\[
d(s) = \left\{ \begin{array}{ll}
d + \epsilon(s) & \text{if } \epsilon(s) < \epsilon(\tilde{s}) \\
K & \text{otherwise}
\end{array} \right.
\]

- For simplicity, assume that the distribution of output is continuous (implying an infinite number of states \( s: S \rightarrow \infty \)). Under this assumption, \( \epsilon(\tilde{s}) \) is defined precisely as \( d + \epsilon(\tilde{s}) = K \).
Asset trade with costly default

Using the fact that $d$ is obviously

$$d = K - \epsilon(\tilde{s})$$

with risk neutral and competitive investors, the expected cash flow is

$$E_t d(s) = \sum_{\epsilon(s) < \epsilon(\tilde{s})} \pi(s)(d + \epsilon(s)) + \sum_{\epsilon(s) \geq \epsilon(\tilde{s})} \pi(s)K =$$

$$= \sum_{\epsilon(s) < \epsilon(\tilde{s})} \pi(s)(d + \epsilon(s)) + \sum_{\epsilon(s) \geq \epsilon(\tilde{s})} \pi(s)(d + \epsilon(\tilde{s})) =$$

$$= d + \sum_{\epsilon(s) < \epsilon(\tilde{s})} \pi(s)\epsilon(s) + \sum_{\epsilon(s) \geq \epsilon(\tilde{s})} \pi(s)(\epsilon(\tilde{s}))$$

and since:

$$\sum_{\epsilon(s) < \epsilon(\tilde{s})} \pi(s)\epsilon(s) = - \sum_{\epsilon(s) \geq \epsilon(\tilde{s})} \pi(s)\epsilon(s)$$
Asset trade with costly default

\[ E_t d(s) = d - \sum_{\epsilon(s) \geq \epsilon(\tilde{s})} \pi(s) (\epsilon(s) - \epsilon(\tilde{s})) = 0 \]

from which

\[ d = K - \epsilon(\tilde{s}) = \sum_{\epsilon(s) \geq \epsilon(\tilde{s})} \pi(s) (\epsilon(s) - \epsilon(\tilde{s})) \geq 0 \]

1. there is a critical level of output, at which the country is indifferent between defaulting and not defaulting, since payments are equal to the cost of the sanction;

2. the constant \( d \) is non-negative.

Note:
- For \( K = 0 \), we are back to the case of no trade.
- For \( K \) large enough, \( d = 0 \) and insurance is perfect. The higher the ability by international markets to impose sanction on the delinquent country, the higher the welfare of domestic residents.
Asset trade with costly default

\[ C(s) = Y \text{ with perfect risk insurance} \]

\begin{align*}
\text{Outflow of Payment} & \quad \text{Inflow of payment} \\
45 & \quad 0 \\
Y + e(s) & \quad Y - e(s) \\
\text{Min} & \quad \text{Max} \\
\end{align*}

\[ \begin{align*}
& \text{Inflow of payment} \\
& \text{Outflow of Payment} \\
& k \\
& d \\
& e(s) - \\
\end{align*} \]
Effect of increasing the cost of default

\( C(s) = Y \) with perfect risk insurance

\[
\begin{align*}
\text{Outflow of Payment} & \quad \text{Inflow of payment} \\
45 & \quad 0 \\
Y + e(s) & \quad Y - e(s) \\
\text{Min} & \quad \text{Max}
\end{align*}
\]

\( k \)
Suppose international investors can punish a defaulting country by excluding it from international financial markets. This means that a default triggers a welfare cost, given by the difference in utility between a trade equilibrium with some risk sharing, and under financial autarky.

For simplicity, consider an infinite-horizon economy version of the economy described in the previous section. Assume further that (a) all contracts are defined at time $t$ with payments $d(s)$ depending exclusively on the realization of the state of nature $s$; and (b) the exclusion from financial trade in case of default lasts forever.
Reputation costs

Welfare under financial trade is

\[ U(Y + \epsilon(s) - d(s)) + \sum_{t=1}^{\infty} \beta \sum_s \pi(s) U(Y + \epsilon(s) - d(s)) = \]

\[ = U(Y + \epsilon(s) - d(s)) + \frac{\beta}{1-\beta} \sum_s \pi(s) U(Y + \epsilon(s) - d(s)) \]

welfare under autarky is

\[ U(Y + \epsilon(s)) + \frac{\beta}{1-\beta} \sum_s \pi(s) U(Y + \epsilon(s)) \]

The incentive compatibility constraint states that the country refrains from defaulting on its external liabilities as long as the former is larger or equal to the latter expression.
Reputation costs

Taking the difference between the above expressions and rearranging

\[ U(Y + \epsilon(s)) - U(Y + \epsilon(s) - d(s)) \leq \]

\[ \frac{\beta}{1 - \beta} \sum_s \pi(s) [U(Y + \epsilon(s) - d(s)) - U(Y + \epsilon(s))] \]

‘Reputation’: the temptation to default (LHS) must be outweighed by the gains of accessing financial markets in the future. Note that the above inequality is more likely to hold the larger \( \beta \), i.e. the more agents value future over current utility, and if \( \epsilon(s) \) is bounded.
There are at least three key features/complications of reputation.

1. The above stipulates that exclusion from financial markets is forever and total. If the ‘exclusion’ be recontracted over time, the cost is obviously lower.

2. It does not work if the horizon is finite. If there is a ‘last period’ $T$, it is apparent that a reputational equilibrium is not possible. The country will always default on payment at $T$. Knowing this, international investors would not sign financial contracts for $T - 1$. But this means that the country will obviously default already in $T - 2$. The argument extends by ‘backward induction’ to period $t$.

3. It makes a difference whether creditors can seize foreign assets by the debtor (a point stressed by Bulow and Rogoff). Intuitively, suppose a country default on payment $d(s)$ but deposit this amount abroad. If the deposit cannot be seized, the country can use this amount as collateral to write insurance contracts/borrow, by-passing the exclusion. But this means that the cost of default can largely be avoided.
Implications for international financial contracts

Above we have discussed the implications of sovereign risk for how much countries can trade across the border. We now discuss the implications for the type of financial contracts that are (likely to be) traded across borders. Denote the repayment schedule promised by the country residents with \( x(y) \), to stress that cash flows will in principle be contingent on the state of the economy \( y \). We now draw on theory to gain insight on the implication of sovereign risk for the type financial contracts that a likely to be traded across markets. Our focus is on whether:

1. \( y \) is verifiable, i.e. ex-post creditors and/or a third party can “tell what \( y \) is”;
2. contracts can be enforced — there is some ‘authority’ that can force a country to honour its liabilities.

Contrast 3 cases:

1. output \( y \) is verifiable and contracts are enforceable
2. output \( y \) is verifiable and contracts are not enforceable
3. output \( y \) is not verifiable and contracts are not enforceable
Implication for international financial contracts

Case 1: if \( y \) is verifiable and contracts are enforceable, the debtor cannot but keep his/her promises. The amount a country can obtain from international investors in period 0 is only constrained by its ‘ability to pay’ in period 2:

\[
x(y) \leq y
\]

- In this world, it is easy to write contracts with a variable payment schedule \( x(y) \) contingent only on realization of economic fundamentals.
- With payments rising or falling with good and bad realizations of \( y \), contracts look like ‘equities’. The country can share output risk with creditors.
Case 2: output is verifiable and contracts are not enforceable
If no international authority can force a country to honour its contracts, the relevant constraint is ‘willingness-to-pay’.

- Trade is possible only if default entails some “costs” that discourage the country from reneging on its international obligation.
  - The country can be expected to honour financial contracts up to
    
    \[ x(y) \leq K = \text{costs of default} \]

- Conditional on \( K \), if \( y \) is verifiable, payments \( x(y) \) can still be made conditional on it: debt contracts may still look a bit like ‘equities’. But parties understand that payments will not exceed \( K \).
Case 3: output is not verifiable and contracts are not enforceable

To the extent that creditors and/or international organizations cannot verify the true economic performance of a country, the country will have a strong incentive to misrepresent reality—e.g. it may claim to be in a poor state, and unable to pay the contractual interest bill, when, in reality, its economy is not doing as bad.

A large body of research in economics suggests that lenders will set the terms of financial contracts — amount of credit and payment schedule — to provide incentives to the country to ‘reveal the truth’. These contracts may look like non-contingent debt $b_0$ studies above, i.e., the cash flow written in the contracts will generally not be contingent on economic circumstances ($x^H = x^L = x$). Yet, as in case 2, country will never repay in excess of default costs $x \leq K$. 
Consequences for int’l asset trade and risk sharing

1. Financial contracts tend to take the form of non-contingent debt—as a result, cross-border risk sharing is not efficient.
   - Note that in our initial example, since debt is state contingent, the agents who default are the “lucky ones”, i.e., default occurs when income is high. When debt is not state contingent, however, the incentive to default is high also under economic distress, whereas raising resources for repayment produces higher disutility and/or may be more costly.

2. Countries are credit-constrained
   - They would like to borrow based on their ‘ability to pay’, but they cannot, since creditors do not trust their commitment to honour their liabilities.
Appendix: Consumption smoothing with non-contingent debt

**Non contingent debt:** In our introductory example, suppose the country residents can issue non-contingent debt, i.e., paying the same interest rate $R$ independently of the ex-post realization of their individual income. Let $b_0$ denote individual level borrowing, and $x_1$ the interest bill $Rb_0$ in period 1. Then

$$\text{Max } U(c_{i,0}) + EU(c_{i,1})$$

$$c_0 = y_{i,0} + b_0 \text{ and } c_{i,1} = \begin{cases} c^H_{i,1} = y^L_{i,1} - x_1 \\ c^L_{i,1} = y^L_{i,1} - x_1 \end{cases}$$

whereas, because (a) $R = 1$ and (b) international investors are risk neutral and do not discount the future, $b_0$ will be equal to the expected payment:

$$b_0 = \frac{1}{2}x_1 + \frac{1}{2}x_1 = x_1$$

As long as the payment on $b_0$ are non-contingent, (a) issuing debt will not allow the country residents to smooth consumption completely; (b) uncertainty (a mean preserving spread of individual income) will raise precautionary saving.
Consumption smoothing with non-contingent debt

The optimal borrowing problem is

\[ \text{Max } U(y_{i,0} - b_0) + \left[ \frac{1}{2} U(y_{i,1}^H - x_1) + \frac{1}{2} U(y_{i,1}^L - x_1) \right] \]

the first order condition is:

\[ U'(0.5 + x_1) = \frac{1}{2} U'(2 - x_1) + \frac{1}{2} U'(1 - x_1) \]

To wit, consider \( U(C) = \ln(C) \). The first order condition

\[ \frac{1}{(0.5 + x_1)} = \frac{1}{2} \left[ \frac{1}{2 - x_1} + \frac{1}{1 - x_1} \right] = 0 \]

is solved by \( b_0 = x = 0.388 \). Consumption will be

\[ c_0 = y_{i,0} + b_0 = 0.5 + 0.39 = 0.89 \quad \text{and} \quad c_{i,1} = \begin{cases} c_{i,1}^H &= y_{i,1}^H - x_1 = 2 - 0.39 = 1.61 \\ c_{i,1}^L &= y_{i,1}^L - x_1 = 1 - 0.39 = 0.61 \end{cases} \]

Welfare is higher than under financial autarky (to wit: verify that with log preferences, expected utility is \(-0.126\) with a bond and \(-0.347\) under financial autarky). But the allocation is not efficient. Note that consumption when income is low in period 1 is quite low. The agents consumption is exposed to income risk.